1 Main information

Definition 1.
Triangle $ABC$ is called similar to triangle $A_1B_1C_1$ (denoted by $ABC \sim A_1B_1C_1$) if and only if one of the following equivalent conditions is satisfied:

- **SSS** $AB : BC : CA = A_1B_1 : B_1C_1 : C_1A_1$;
- **SAS** $AB : BC = A_1B_1 : B_1C_1$ and $\angle ABC = \angle A_1B_1C_1$;
- **AA** $\angle ABC = \angle A_1B_1C_1$ and $\angle BAC = \angle B_A_1C_1$.

Theorem 1 (Thales or intercept theorem).
Suppose $S$ is the intersection point of two lines and $A$, $B$ are the intersections of the first line with the two parallels, such that $B$ is further away from $S$ than $A$, and similarly $C$, $D$ are the intersections of the second line with the two parallels such that $D$ is further away from $S$ than $C$. Then $SAC \sim SBD$.

2 Problems

Definition 2.
A middle line of a triangle is the segment that connects the midpoints of two sides.

Problem 1. (a) Draw a triangle, then draw all three middle lines of the triangle.

(b) Prove that a middle line is parallel to the third side and is equal to half its length.

Problem 2.
In an acute-angled triangle $ABC$ the altitudes $AA_1$ and $BB_1$ are drawn. Prove that $A_1C \cdot BC = B_1C \cdot AC$.

Problem 3.
Squares $ABCD$ and $AEFG$ share a vertex $A$. Prove that a) $ABE = ADG$;
b) $ACF \sim ABE$. 
Problem 4.
Prove that the medians of a triangle intersect at one point and are divided by this point in a ratio of $2:1$, counting from the vertex.

Problem 5.
Using ruler and compass, divide a given segment in a ratio of $5:7$.

Problem 6.
The square $PQRS$ is inscribed in the triangle $ABC$ so that the vertices $P$ and $Q$ lie on the sides $AB$ and $AC$, and the vertices $R$ and $S$ lie on the side $BC$. Express the length of the side of the square in terms of side $a$ and the height $h_a$.

Problem 7.
Prove that the midpoints of the sides of an arbitrary quadrilateral are the vertices of a parallelogram. For which quadrilaterals is this parallelogram a rectangle, for which is it a rhombus, for which is it a square?