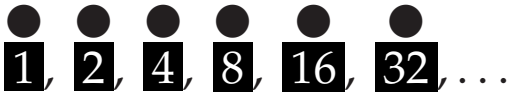
Weighing with Powers of 2

In the country of Binary Land, the factory makes only the following weights (in grams):



You can buy any number of any of these standard weights from the factory and use them with a balance scale.

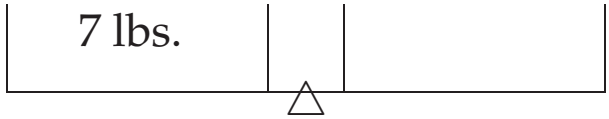
1. How are the weights in the sequence above related?

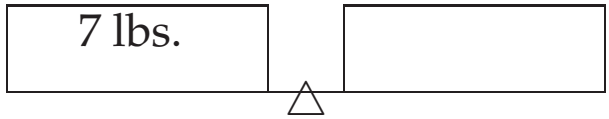
2. Write down the next several weights in the sequence.

3. Balance each of the following objects using the weights

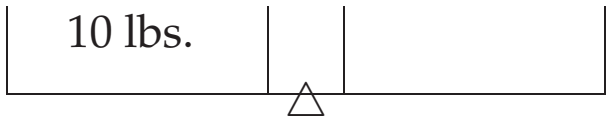
(You have many copies of each of the weights):

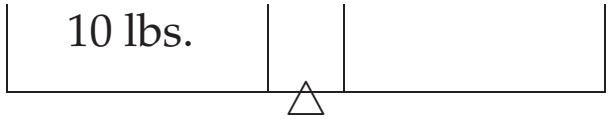
(a) A watermelon weighting 7 pounds: (find at least two different ways to balance it using the given weights).



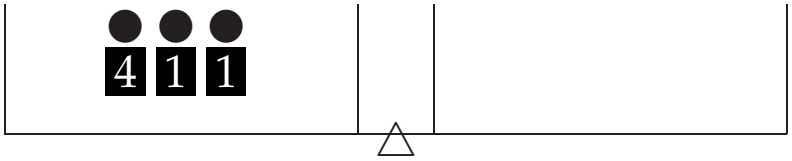


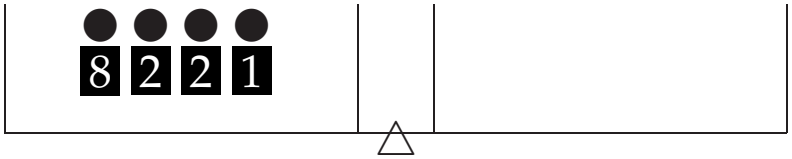
(b) A metal ball weighing 10 pounds (find at least two different ways to balance it using the given weights).

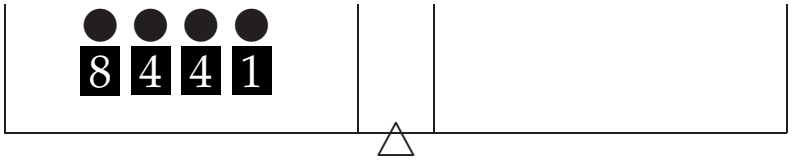


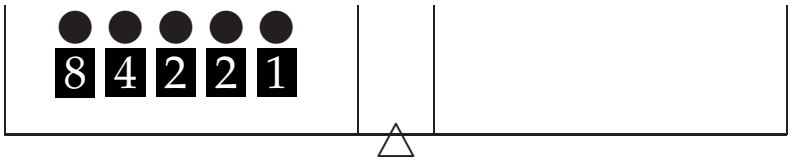


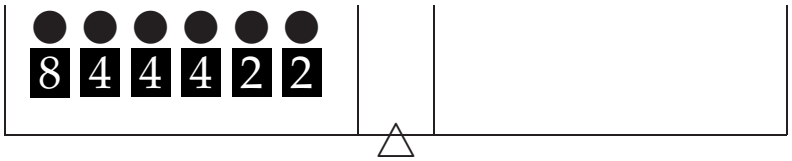
4. Now try to use each of the standard weights only once. Can you balance the following:

(a) 

(b) 

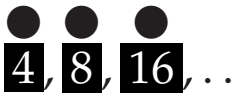
(c) 

(d) 

(e) 

5. Someone stole the 1 gram weight. What kind of weights can you balance now? (You still have all of the weights 

, . . .).

6. The next day, the 2 gram weight got stolen, too. Which weights can you balance now? (You have the weights 

, . . .).

7. They found the 1 gram weight! Which weights can you balance now? (You have the weights 

, . . .).

8. Balance the following. Do not use the same weight more than once. Fill in the table below:

   - if you are using a certain weight, put 1 in its column;
   - if you are not using a certain weight, put 0 in its column.

   Some examples are filled in:

Weight	Write as sum of weights	8	4	2	1
1	$1 =$ 1	0	0	0	1
2	$2 =$ 2	0	0	1	0
3	$3 =$? ?				
4	$4 =$?	0	1	0	0
5	$5 =$				
6	$6 =$ 4 2	0	1	1	0
7	$7 =$				
8	$8 =$				

Weight	Write as sum of weights	8	4	2	1
9	$9 =$ 8 1	1	0	0	1
10	$10 =$				
11	$11 =$				
12	$12 =$				
13	$13 =$ 8 4 1	1	1	0	1
14	$14 =$				
15	$15 =$				

9. Major robbery!

   (a) After many weights were stolen you are left only with the weights 1, 2, 4 and 8. Can you balance 18 grams using each weight no more than once? Why or why not? What weights can you balance?

   (b) What is the largest number you can write as a sum of some the numbers $1, 2, 4, 8$ without using the same number more than once?

(c) Can you write down 30 as sum of some of the numbers

$$1, 2, 4, 8, 16, \ldots$$

without using any number more than once? That is,

$$30 = \ldots + \ldots + \ldots + \ldots$$

Now write down the string of 0s and 1s that corresponds to 30:

	16	8	4	2	1
30					

(d) Can you write down 57 as sum of some of the numbers

$$1, 2, 4, 8, 16, 32, \ldots?$$

Do not use the same number twice.

$$57 = \ldots + \ldots + \ldots + \ldots$$

Now write down the string of 0s and 1s that corresponds to 57:

	32	16	8	4	2	1
57						

10. What number corresponds to the following strings of 0s and 1s. Fill in the table:

32	16	8	4	2	1	computation	number
0	1	1	0	1	0	$16+8+2=26$	26
0	1	1	0	1	0		
0	0	1	1	0	1		
1	1	0	0	1	0		
1	1	1	1	1	1		

11. *Every number can be written in binary notation:*

$1 = \boxed{1}$, $2 = \boxed{1}\boxed{0}$, $3 = \boxed{1}\boxed{1}$, $4 = \boxed{1}\boxed{0}\boxed{0}$.

Find the missing numbers or binary notation:

$\boxed{1}\boxed{0}\boxed{1} =$ $\square\square\square = 6$ $\quad$ $\boxed{1}\boxed{1}\boxed{0} =$ $\square\square\square = 7$ $\quad$ $\boxed{1}\boxed{0}\boxed{0}\boxed{1} =$ $\square\square\square = 8$

12. Explain the following addition problem:

$$\begin{array}{rccc} & \cdot & \dot{5} & 4 \\ + & & 4 & 6 \\ \hline & 1 & 0 & 0 \end{array}$$

Now look at a similar problem in binary notation. Convert to decimal numbers and check!

$$\begin{array}{rcccc} & \dot{\square} & \dot{1} & 1 & \\ + & & & 1 & \\ \hline & 1 & 0 & 0 & \end{array}$$

13. Perform the following additions of binary numbers:

$$\begin{array}{rcccc} + & & 1 & 1 & \\ & 1 & 0 & 1 & \\ \hline & \square & \square & \square & \square \end{array} \qquad \begin{array}{rcccc} + & 1 & 0 & 1 & \\ & 1 & 0 & 1 & \\ \hline & \square & \square & \square & \square \end{array}$$

14. Compute the sum $1 + 1 + 2 + 4 + 8 + 16 + 32$ like this: first, convert all numbers to binary. Then carry out the addition. Last, convert to a decimal number.

$$\begin{array}{rcccccc} + & \square & \square & \square & \square & \square & \square \\ + & \square & \square & \square & \square & \square & \square \\ + & \square & \square & \square & \square & \square & \square \\ + & \square & \square & \square & \square & \square & \square \\ + & \square & \square & \square & \square & \square & \square \\ + & \square & \square & \square & \square & \square & \square \\ + & \square & \square & \square & \square & \square & \square \\ & \square & \square & \square & \square & \square & \square \\ \hline \square & \square & \square & \square & \square & \square & \square \end{array} = \underline{\quad\quad}$$