Some Word Problems

1. Five friends sit around a circular table to eat lunch together. Ms. Ostrich is sitting between Ms. Meany and Ms. Markey. Cheryl is sitting between Linda and Ms. Rodriguez. Ms. Meany is sitting between Cheryl and Nancy. Linda and Genevieve are sisters. Maria is seated with Ms. Ducky on her left and Ms. Markey on her right.

   What are the names of the five people sitting at the table?

2. Farmer Roy ties his pet iguana, Steve, outside to eat grass.

   (a) If Steve is tied to the corner of a rectangular barn that is 4 meters by 5 meters long, with a rope that is 6 meters long, how much grazing area does he have?

   (b) If Steve is tied to the corner of a strange barn that is an equilateral triangle, each side 5 meters long, with a 7 meter rope how much grazing area is available?

3. Ian has a strange desk calendar. His calendar consists of two wooden cubes. Each cube has one digit from 0-9 on each of its faces. Each day, you flip and rotate the cubes so that the two-digit day of the month (from 01 to 31) appears. How can this be the case?

4. Suppose you take a long strip of paper and fold it in half at the middle. Then you fold it again, and again, and again ... After 15 folds, how many creases are there in the paper. (Note: It's not physically possible to fold a piece of paper in half 15 times, but just suppose.)

Super Secret Spy Stuff

A nefarious, but somewhat incompetent intelligence service uses a code in which the digits $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$ are replaced by capital letters. There is a different code for each day of the week, but in each code it is well known that every digit is represented by one and only one letter.

1. If the code says $A = 9$, $B = 2$, $E = 3$, $H = 0$, $M = 1$, $N = 7$, $O = 4$, $S = 5$, $T = 6$, and $Y = 8$, then what does YES + NO equal?
2. Can you find a code which makes the equation FLIP + FLOP = SHOE correct?
3. If the Tuesday code says that LOVES + LIVE = THERE, what is the *maximum possible* value of THERE?
4. (*) A code intercepted on Thursday says BLASE + LBSA = BASES. Your agent examines the code and tells you there are two possible decryptions. Is he right?

The Four Color Theorem

In 1852, the South African mathematician Francis Guthrie noticed when coloring a map of counties of England that he could always manage to use just four colors. The **four color theorem**, loosely stated, says that this is always the case. Whenever you draw a map on a globe or a piece of paper, you never need more than four colors to shade in all regions on the map in such a way that no two adjacent regions are the same color.

Some important clarifications:

- **Each region must be one connected piece.**

  In maps of the world, some countries (such as the USA) have multiple unconnected pieces which more properly ought to have the same color.

- **Two regions are adjacent only if they have some positive length of boundary.**

  Thus if two regions meet only at a corner they are not considered adjacent.

If we don't assume *both* of these conditions, the theorem does not apply.

That you only need four colors is an easy enough pattern to spot when you try coloring lots of maps, but just trying many examples does not constitute a proof. For one thing, there are infinitely many possible maps. For another, if you start coloring a map with four colors, depending on how you start, it may or may not be possible to finish with just four colors. Sometimes an early decision turns out to be disastrous! This makes it very hard to reduce the problem to simple cases.

Many people tried to prove this fact, but for over a hundred years, each of them turned out to have made a mistake in their purported proof. It wasn't until the 1970's that mathematicians were able to finally prove it, and then only with the help of computers!

You can read more at `http://en.wikipedia.org/wiki/Four_color_theorem`, for example.

Some Map Coloring Problems

1. (a) Draw a map that can't be colored with three colors. This proves that there *isn't* any "three color theorem" waiting to be proven.

   (b) Let us say that a group of regions form a "clique" if each of these countries shares a border with every other country. Try to do the first problem again, but in such a way that there are no "cliques" with four countries.

2. Can you draw a "map" where you need at least *five* colors to color all the countries? How this map have to be different from the normal kind?

3. (HARD) Suppose you assign a color to *every* point in the plane, such that if two points are a distance 1 apart, then they cannot be the same color. One could ask what the smallest number of colors needed is. The answer to this question is not known! It is either 4,5,6, or 7.

   Can you explain why three colors isn't enough, and why seven is?