Probability and Independence

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NOTES:

We say two events A and B are **independent** if $P(A \text{ and } B) = P(A) \times P(B)$.

The **Inclusion-Exclusion Principle** states that for any events A, B, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

1. One die is rolled twice. Find the probability of each of the following events, and of each *pair* of events (e.g. for event A, B find $P(A)$, $P(B)$ and $P(A \text{ and } B)$)

   (A) A two is rolled on the first roll.

   (B) A two is rolled on the second roll.

   (C) The sum of the two rolls is six.

   (D) The sum of the two rolls is seven.

   Which pairs of events are independent? Why? (Hint: To justify your answer, you have to use the definition of independence.)

2. Flopsy the Turtle rolls three fair dice.

   (a) What is the probability that all three dice come up the same?

   (b) What is the probability that at least two of the dice come up the same?

   (c) What is the probability of rolling three different numbers?

3. Say you flip a coin ten times.

   (a) What is the probability of getting at least one head?

   (b) At least two heads?

   (c) At least ten heads?

4. A certain bag contains three red balls, three green balls, and six yellow balls.

   (a) What is the probability of drawing a yellow ball? What is the probability of drawing a green ball?

   (b) Julia draws two balls without replacement. What is the probability that she draws a yellow ball and a green ball?

   (c) Now Julia draws two balls *with* replacement. What is the probability that she draws a yellow ball and a green ball?

   (d) What is the difference (from the point of view of independence) of drawing with replacement and drawing without replacement?

5. A certain event A is independent of itself. What could $P(A)$ be?