Geometry Welcome*
(LAMC, Fall 2008)

September 20, 2008

We ask all LAMC students who have already studied Geometry to work on the problems below and send us your solutions before by **September, 13th.** This will give us a week before the first fall meeting to grade your solutions very carefully and determine the winners.

In order to make LAMC sessions the best for you, we need to know as much as possible about each of you individually and as a group. By submitting your solution, you are introducing your mathematical self to us. Please work on the problem set even if you have been participating in the LAMC before. **The top scorers will receive prizes (math books and cool certificates!)**

There are two ways to send your solutions:

1. By e-mail: send to radko@math.ucla.edu with the subject "Geometry Welcome solutions"
2. By regular mail to
   Olga Radko
   Department of Mathematics
   UCLA,
   Los Angeles, CA, 90095

Here are some basic rules:

1. The work should be your own. You should not receive help from teachers, friends, parents, etc.
2. You should justify all your steps and prove all of your statements.
3. Please do not search for solutions on the internet. We want to see how you can do problem solving as opposed to how you can use search engines. (Not to mention that such instances are often easy to detect).
4. Make sure you include your name and grade.

Good luck and have fun!

*Please write to Olga Radko at radko@math.ucla.edu if you have any questions on the problems.

1. Prove that in any triangle a median drawn to a side is smaller than half of the sum of the other two sides.
2. Two circles of radii R and r respectively are tangent to each other. A line l is tangent to both circles, at points A and B respectively. Find the length of the segment AB in terms of R and r.
3. In $\triangle ABC$ $|AC| = 6cm$, $|BC| = 4cm$ and $\angle B = 2\angle A$. Compute $|AB|$.
4. Prove that if two medians in a triangle are congruent, then the triangle is isosceles.
5. Find the area of an isosceles trapezoid with perpendicular diagonals and given length of midline.
6. Recall that **rotation** with given center O by angle α moves any point X on the plane to the point X' such that $|OX| = |OX'|$ and $\angle X'OX = \alpha$. Perform two rotations in a row. The first rotation has center A and is by angle α. The second rotation has center B and is by angle β. Prove that this is the same as just one rotation. Find the center and the angle of this new rotation.
7. There are 6 points inside of the rectangle of size 3×4. Show that among these points there are two such that the distance between them is no bigger than $\sqrt{5}$.
8. Is it true that if all the sides of a triangle are less then 1 then the area is less than $\sqrt{3}/4$?
9. Let A and B be two cities in the Flatland. (Flatland is an infinite plane). Find the set of all points M such that if you start going from M to A along the straight line connecting these points, the distance between you and point B always increases.
10. Let ABC be a triangle and M be a point inside of the triangle. Draw a line segment with end points on the sides of the traingle in such a way that M is the midpoint of this segment. (Here "draw" means that you need to construct the endpoints of the segment using a compass and a ruler only).