# Problem Solving Welcome
# (LAMC, Fall 2008)

September 20, 2008

We ask all LAMC students who did *not* yet have Geometry to work on the problems below and send us your solutions before by **September, 13th.** This will give us a week before the first fall meeting to grade your solutions very carefully and determine the winners.

In order to make LAMC sessions the best for you, we need to know as much as possible about each of you individually and as a group. By submitting your solution, you are introducing your mathematical self to us. Please work on the problem set even if you have been participating in the LAMC before. **The top scorers will receive prizes (math books and cool certificates!)**

There are two ways to send your solutions:

1. By e-mail: send to radko@math.ucla.edu with the subject "Geometry Welcome solutions"

2. By regular mail to
   Olga Radko
   Department of Mathematics
   UCLA,
   Los Angeles, CA, 90095

Here are some basic rules:

1. The work should be your own. You should not receive help from teachers, friends, parents, etc.

2. You should justify all your steps and prove all of your statements.

3. Please do not search for solutions on the internet. We want to see how you can do problem solving as opposed to how you can use search engines. (Not to mention that such instances are often easy to detect).

4. Make sure you include your name and grade.

**Good luck and have fun!**

1. A team of bakers consists of an experienced chef and 9 student bakers. During the day each student baker decorated 15 cakes. The number of cakes decorated by the chef is 9 more then the team's average.
How many cakes were decorated by the whole team on this day?

2. Find all $x$ such that $x^{x^3} = 3$. (Here $x$ is raised to the power $x^3$).

3. At noon both the hour and the minute hands of an analog clock are pointing up, and thus coincide. When will the two hands be pointing in the same direction (coincide) next time?

4. Without using a calculator, decide which number is bigger: $\sqrt[2]{2}$ or $\sqrt[5]{5}$?

5. My favorite 3 digit number is such that if you subtract 7 from it, the result is divisible by 7; if you subtract 8 from it, the result is divisible by 8; if you subtract 9 from it, the result is divisible by 9. What is my favorite 3 digit number?

6. What is the remainder that you get when the number $13^{16} - 2^{25} \cdot 5^{15}$ is divided by 3? Can you find the remainer of division of the same number by 37?

7. A farmer sold half of his apples and half of an apple to the first customer; after that he sold half of the remaining apples plus another half of an apple to the second customer; etc. After he sold half of the remaining apples and half of an apple to the 7th customer, there were no more apples left. How many apples did the farm have in the beginning?

8. Consider the sequence of numbers:

$$1,\ 3,\ 7,\ 15,\ 31,\ ...$$

   (a) Determine how each next number is obtained from the previous one.

   (b) Find the 10th term.

   (c) Find the 100th term.

9. The product of two positive numbers is equal to 100.

   (a) How small can the sum of these numbers be?

   (b) How large can the sum of these numbers be?

   (c) What are the answers to the questions above if both numbers are required to be integers (whole numbers)?

10. Compute without using a calculator:

$$20^2 + 19^2 + \ldots + 11^2 - 10^2 - 9^2 - \ldots - 1^2$$