# Parity

Parity is the property of an integer to be even (i.e. divisible by 2) or odd (i.e. not divisible by 2). For example, $6 = 2 \cdot 3$ is an even number and $7 = 2 \cdot 3 + 1$ is an odd number.

If among two numbers both are even or both are odd they are said to have the same parity. For example, 6 and 14 have the same parity; also, 7 and 11 have the same parity. But 5 and 12 have opposite parity.

Parity comes up in a variety of problems in different areas of mathematics.

1. Prove that the sum of two odd numbers is even and the product of two odd numbers is odd.

2. Can you find 7 odd numbers whose sum is 100?

3. The product of 22 integers (whole numbers, including negative numbers) is equal to 1. Show that their sum cannot be 0.

4. The numbers 1 through 10 are written in a row. Can the signs "+" or "-" be placed between them, so that the value of the resulting expression is 0?

5. Given two integers a and b, consider the expression $(a - b) \cdot a \cdot b$. Can you determine if it is even or odd?

6. Show that if the sum of two numbers is odd then their product is even.

7. Ada has several pennies, nickels, and dimes in her savings. She took 10 coins and counted their total, which happened to be 25 cents. Show that at least one of the coins Ada took is a dime.

8. A group of kindergarten students (with boys and girls) dances in a circle in such a way that for every child, both neighbors are of the same gender. Given that there are 10 boys in the circle, how many girls are there?

9. Eleven gears are placed on a wall, arranged in a circular chain. Can all the gears rotate simultaneously?
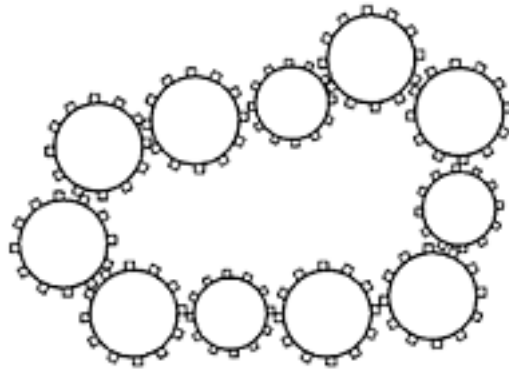
10. John writes the numbers 1 through 49 on paper cards. After that, he turns the cards over, mixes them up and writes the same numbers on the other side. For every card, John adds the numbers written on both sides and then multiplies all these sums. Show that the number he gets is even. (Hint: What would it mean if the number he got was odd?)