

Junior Circle- More fun and games!

1. Chris and Liz are playing a game where they take turns writing down the digits 1 through 8 to form an 8-digit number:

- First Chris writes a number, then Liz writes a number, and so on;
- Each digit can be used only once;
- The game ends when all the digits are used;
- Liz wins if the number at the end of the game is even. Chris wins if the number at the end is odd.

a. Find a partner and play this game. Someone will be player 1 (like Chris) and the other will be player 2 (like Liz). Write the numbers chosen by each player below.

Player 1	Player 2	Player 1	Player 2	Player 1	Player 2	Player 1	Player 2	Winner?

b. Now switch roles and let player 2 now be player 1 and vice versa. Write the numbers chosen by each player below.

Player 1	Player 2	Player 1	Player 2	Player 1	Player 2	Player 1	Player 2	Winner?

c. If Chris is player 1 and Liz is player 2, who can always win and how?

2. There are several pennies sitting in a row on a table. Chris and Liz alternate taking pennies from the table. The player who takes the last penny (or pennies) wins.
- a. Suppose there are 8 pennies. First Chris and Liz play the game where they are only allowed to take one penny at a time. If Chris goes first and Liz goes second, who will always win?

 - b. Now Chris and Liz are allowed to take either one OR two pennies per turn.
 - i. If there are just 3 pennies in the beginning, how many pennies should the first player take? Will he always lose?

 - ii. If there are 4 pennies in the beginning, how many pennies should the first player take? Can he always win?

iii. If there are 5 pennies in the beginning, how many pennies should the first player take? Can he always win?

iv. If there are 6 pennies in the beginning, how many pennies should the first player take? Will he always lose?

v. If there are 7 pennies in the beginning, how many pennies should the first player take? Can he always win?

vi. If Chris goes first and there are 8 pennies, how many pennies should he take? Can he always win?

3. I have four cards with numbers 1 through 15 on them. (The same number appears on several cards). Here are the cards (numbered card #1 through card #4 from left to right):

$$\begin{pmatrix} 8 & 9 & 10 & 11 \\ 12 & 13 & 14 & 15 \end{pmatrix} \quad \begin{pmatrix} 4 & 5 & 6 & 7 \\ 12 & 13 & 14 & 15 \end{pmatrix} \quad \begin{pmatrix} 2 & 3 & 6 & 7 \\ 10 & 11 & 14 & 15 \end{pmatrix} \quad \begin{pmatrix} 1 & 3 & 5 & 7 \\ 9 & 11 & 13 & 15 \end{pmatrix}$$

- a. As a group, pick a number between 1 and 15, but do not tell your group leader (Shannon, Aaditya, Rosalie, Melanie, Tim, Daniel, or Nikki). Once you've decided on a number, tell your group leader which cards (1 though 4) your number is on. Now, your group leader will guess which number you picked.
- b. Try it again with another number. Did your group leader guess correctly?
- c. Now let's try to figure out the trick. Write the numbers 1 through 15 in binary below:
1=
2=
3=
4=
5=
6=
7=
8=
9=
10=
11=
12=
13=
14=
15=

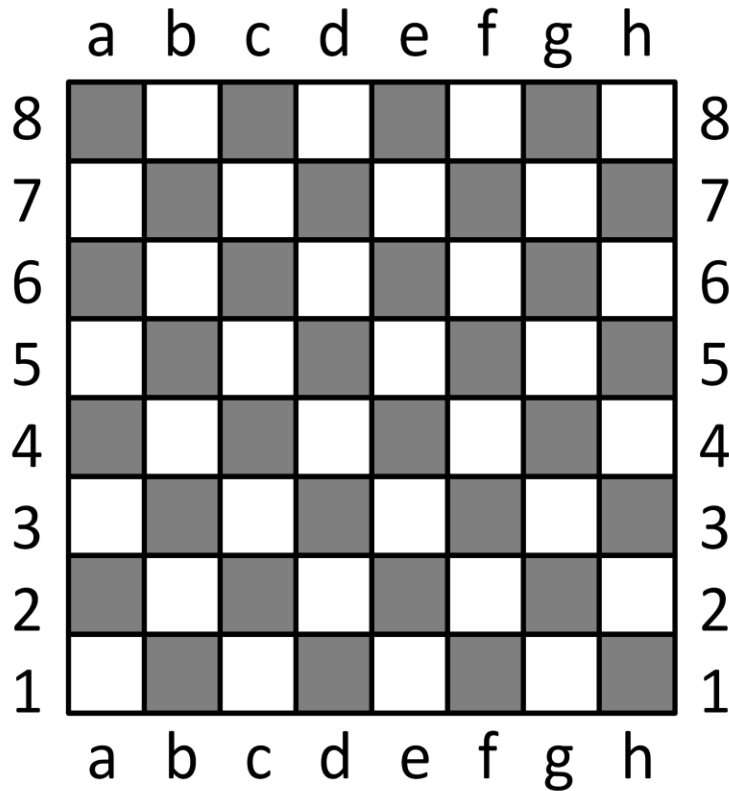
d. What do the numbers on the first card have in common when we look at them in binary?

e. What do the numbers on the second card have in common when we look at them in binary?

f. What do the numbers on the third card have in common when we look at them in binary?

g. Can you explain how the group leader correctly guessed your number?

4. Here is the chessboard we used last time:



We want to find out the fastest path a king can take from a given square to a1. In other words, we want to know the path that takes the smallest number of moves.

- a. If the king is placed on a3, what is the fastest path to a1? How many moves did you make? Is there another fastest path?

- b. If the king is placed on c3, what is the fastest path to a1? How many moves did you make? Is there another fastest path?

- c. If the king is placed on e8, what is the fastest path to a1? How many moves did you make? Is there another fastest path?

5. A man is walking on a road that forks into two roads, one road going to the City of Truth and the other road going to the City of Lies. He wants to go to the City of Truth, but he does not know which road leads to which city. He sees a man (who supposedly came from one of the cities) at the fork as he approaches it. He can only ask that man one question to figure out which city is the City of Truth. What question should he ask him?