

## Combinations Revisited

1. Everyone is excited, because this weekend is the big Pullington County Tug-of-War Contest! Coach Ropybones is making preparations.
  - (a) Coach needs to choose which 5 players from his 8-player team will play in the first match. How many ways are there for him to choose 5 starting players out of the 8?
  
  
  
  
  
  
  
  
  
  
  - (b) Oops, Coach almost forgot—he definitely wants his star player, Rocky, to play in the first match. How many ways are there for Coach to choose the 5 starting players out of 8, provided that one of the 5 must be Rocky?
  
  
  
  
  
  
  
  
  
  
  - (c) Tragedy! The day before the big meet, Rocky has taken gravely ill—he won't be able to play! How many ways are there for Coach to choose 5 starting players out of the remaining 7 team members?
  
  
  
  
  
  
  
  
  
  
  - (d) OK, Coach Ropybones has chosen Ariel, Belle, Jasmine, Pocahontas, and Mulan to play in the first match. Now he needs to decide what order they should pull in, from front of the rope to back. How many different orders are possible?
  
  
  
  
  
  
  
  
  
  
  - (e) Oops, Coach almost forgot—Ariel and Pocahontas are fighting over which one has better hair. He can't put them next to each other or they'll pull on each other's hair instead of the rope. How many ways are there for him to order the 5 players so that there is at least one person between Ariel and Pocahontas?

2. If you flip a fair coin once, you either get heads (H) or tails (T). If you flip the same coin twice, there are four possible sequences—HH, HT, TH, TT. From among these four, there is one sequence with 2 heads (HH), two sequences with 1 head (HT and TH), and one sequence with 0 heads (TT).

Now suppose you flip the coin three times. How many different sequences are there with

- (a) 3 heads?
- (b) 2 heads?
- (c) 1 head?
- (d) 0 heads?

Now flip the coin four times. Again, how many sequences are there with

- (a) 4 heads?
- (b) 3 heads?
- (c) 2 heads?
- (d) 1 head?
- (e) 0 heads?

Once more unto the breach: If you flip the coin 5 times, how many sequences are there with

- (a) 5 heads?
- (b) 4 heads?
- (c) 3 heads?
- (d) 2 heads?
- (e) 1 head?
- (f) 0 heads?

If you flip the coin 50 times, how many sequences are there with 0 heads? 1 head? 2 heads? 3 heads? 48 heads?

3. Calculate...

(a)  $11^0$

(b)  $11^1$

(c)  $11^2$

(d)  $11^3$

(e)  $11^4$

(f)  $11^5$

4. Each morning, Vicki walks from home to school, which is 5 blocks east and 4 blocks south of her home. Vicki can make the trip in many different ways: she could first travel 5 blocks east, and then 4 blocks south. Or, she could go 1 block east, 1 block south, 1 block east, 1 block south, 3 blocks east, and 2 blocks south—or many other ways.

(a) Since variety is the spice of life, she decides she should try to take every reasonable path at least once—“reasonable” of course meaning exactly 9 blocks long, moving one block east or south at each step. How many days will this take, if she tries one new path every day?

(b) Twist! Vicki just realized she needs to stop at the grocery store to get her daily banana on the way to school. The store is three blocks east and one block south of her house. How many different 9-block paths can she take from her house to school, that pass through the grocery store?

(c) Now Vicki's curious. She wants to know, for each intersection east or south of her house, how many possible ways there are to get to that intersection from her house. Label the map below—on each intersection, write the number of paths Vicki can take to get to that intersection. (Hint: It might help to start by labeling intersections at the top and left edges of the map, and then work your way down and to the right from Vicki's home.)

5. \* Compute the sum of the first few rows of Pascal's triangle. Is there a pattern? Can you prove it?

6. \* Prove that  $\binom{n-1}{k-1} + \binom{n-1}{k} = \binom{n}{k}$  for all positive integers  $n$  and  $k$ . (Hint: Think of these as “number of ways to choose items from a group;” look back at problem 1, (a), (b), and (c).)