Geometry of Triangles - I:
Inscribed, Circumscribed, Escribed Circles

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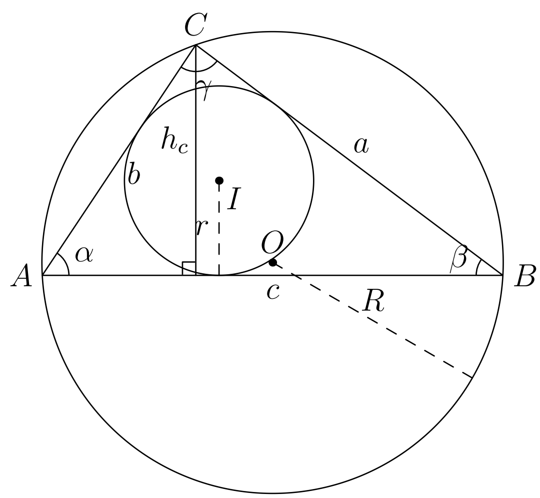

- R — radius of circumscribed circle
- r — radius of inscribed circle
- O — center of circumscribed circle
- I — center of inscribed circle
- h_a, h_b, h_c — altitudes to sides a, b, c
- $S = \frac{1}{2}h_a \cdot a = \frac{1}{2}h_b \cdot b = \frac{1}{2}h_c \cdot c$ — area of the triangle
- $p = \frac{a+b+c}{2}$ — semi-perimeter

Escribed circle is a circle tangent to one of the sides and the extensions of the two other sides. A triangle has 3 escribed circles.

Problems:

1. Prove that BD is a bisector of angle $\angle B$ in $\triangle ABC$, then

$$\frac{AD}{DC} = \frac{AB}{BC}.$$

2. Show that the three angle bisectors intersect in one point.

3. Show that

   (a) the perpendicular bisectors intersect in one point.

   (b) the altitudes intersect in one point (you may use part (a)).

4. Prove the formulas for the radius of circumscribed and inscribed circles:

   (a) $R = \frac{a}{2\sin\alpha}$.

   (b) Show that $r = \frac{S}{p}$.

5. Show that in a right-angle triangle the sum of diameters of inscribed and circumscribed circles equals to the sum of the two shorter sides.

6. Let O be the center of the inscribed circle. Show that $\angle BOC = \frac{\angle BAC}{2} + 90^0$.

7. Let H be the point of intersection of altitudes. Let H_a, H_b and H_c be the points symmetric to H with respect to sides a, b, c respectively. Show that the points H_a, H_b, H_c lie on the circumscribed circle.

8. Show that the three medians intersect in one point. Show that this point divides each median in the ratio $2:1$.

9. Prove that in a parallelogramm the sum of squares of lengths of diagonals equals to the sum of the squares of lengths of sides.

10. Show that a quadrilateral is such that a circle can be inscribed into it iff the sums of the lengths of opposite sides are equal.