

Trigonometry Problems

1. Two rays (a *ray* is a half line) with common endpoint O form a 30° angle. Point A lies on one ray, point B on the other ray, and $AB = 1$. What is the maximum possible value of OB ?

(A) 1 (B) $\frac{1 + \sqrt{3}}{\sqrt{2}}$ (C) $\sqrt{3}$ (D) 2 (E) $\frac{4}{\sqrt{3}}$

2. In $\triangle ABC$, we have $3 \sin A + 4 \cos B = 6$ and $4 \sin B + 3 \cos A = 1$. What is the degree measure of $\angle C$?

(A) 30 (B) 60 (C) 90 (D) 120 (E) 150

3. If $\tan \alpha$ and $\tan \beta$ are the roots of $x^2 - px + q = 0$, and $\cot \alpha$ and $\cot \beta$ are the roots of $x^2 - rx + s = 0$, then rs is necessarily

(A) pq (B) $\frac{1}{pq}$ (C) $\frac{p}{q^2}$ (D) $\frac{q}{p^2}$ (E) $\frac{p}{q}$

4. An object moves 8 cm in a straight line from A to B , turns at an angle α , measured in radians and chosen at random from the interval $(0, \pi)$, and then moves 5 cm in a straight line to C . What is the probability that $AC < 7$?

(A) $\frac{1}{6}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{1}{3}$ (E) $\frac{1}{2}$

5. An isosceles trapezoid is circumscribed around a circle. The longer base of the trapezoid is 16, and one of the base angles is $\arcsin(.8)$. What is the area of the trapezoid?

(A) 72 (B) 75 (C) 80 (D) 90 (E) not uniquely determined

6. What is the value of $\cos 2\theta$ if

$$\sum_{n=0}^{\infty} (\cos \theta)^{2n} = 5 ?$$

(A) $\frac{1}{5}$ (B) $\frac{2}{5}$ (C) $\frac{\sqrt{5}}{5}$ (D) $\frac{3}{5}$ (E) $\frac{4}{5}$

7. In $\triangle ABC$, $\tan(\angle CAB) = \frac{22}{7}$ and the altitude from A divides BC into segments of length 3 and 17. What is the area of $\triangle ABC$?