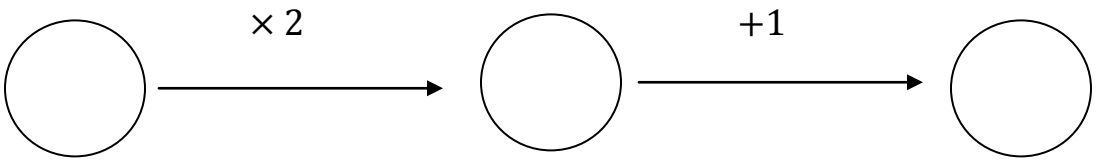
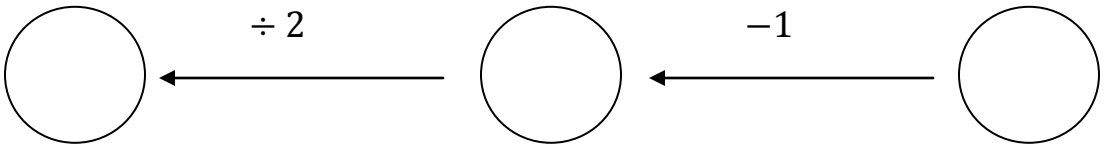
Backwards Reasoning

1. Jane picked a number. She multiplied it by 2 and added 1. She got 15 as a result.
    a. We can draw a picture to show the operations that Jane applied. In the last circle, write the number 15.

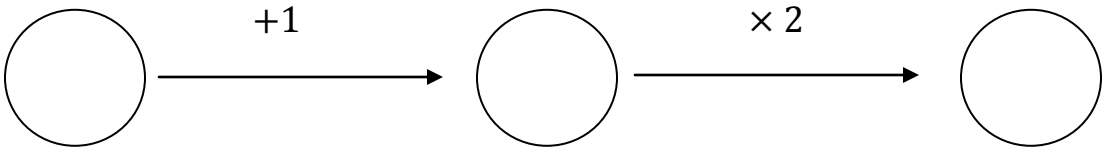


    b. Now, to find Jane's number, we reverse the operations. In each circle, write the number that results from applying the operation.

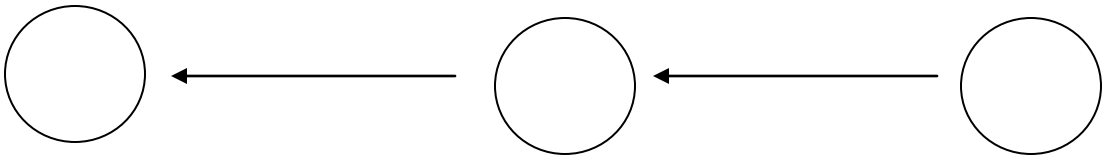


    c. What was Jane's number?

2. Tim took his age, added 1 and multiplied by 2. He got 16.
    a. Again, we can draw a picture to show the operations that Tim applied. In the last circle, write the number 16.



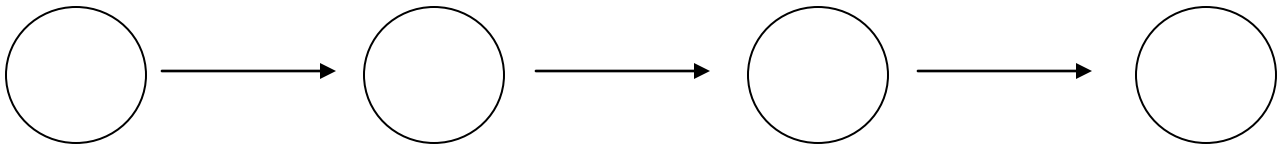
    b. In the picture below, write the reverse operations and in each circle write the number that results from applying the operation.



    c. How old is Tim?

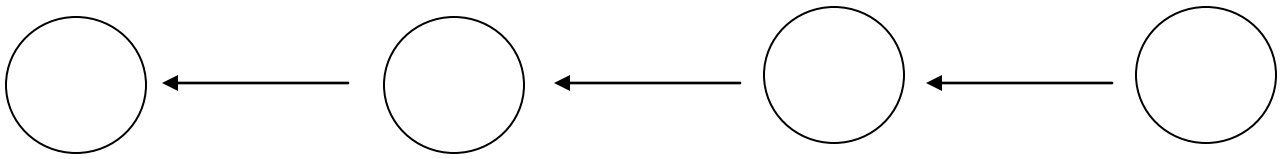
3. In both #1 and #2, the starting number is the same and the operations in the forward direction are “multiply by 2” and “add 1”, but the answers are different. Why is this the case?

4. Kevin and Michelle are cousins. Kevin lives in the United States, while Michelle lives in Europe. Michelle wants to tell Kevin that it is 10 degrees Celsius in her hometown, but in the United States temperature is given in degrees Fahrenheit. She learns that if you divide a temperature in Celsius by 5, multiply by 9, and add 32, you get a temperature in Fahrenheit.
    a. In the picture below, write the operations that Michelle needs to do to convert temperature from Celsius to Fahrenheit.

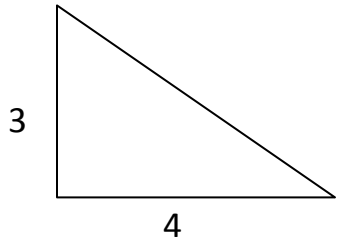


    b. If we start with 10 degrees Celsius in the picture above, what is our answer, in degrees Fahrenheit?

5. Now Kevin wants to tell Michelle that it is 68 degrees Fahrenheit in his hometown. Write the reverse operations in the picture below. What number should Kevin tell Michelle, in degrees Celsius?



6. Gina has a triangle with sides 3 centimeters and 4 centimeters, but she doesn't know the third side. John knows all the sides of the triangle. He tells Gina that if she squares the sides she has and adds them together, she will get the square of the third side. What is the length of the third side of the triangle?



To solve this, let's fill out the table below.

First side	3
Square of first side	
Second side	4
Square of second side	
Square of third side	
Third side	

Challenge problem: Can you think of any other triple of numbers that satisfy this rule?

7. Ann wants us to guess her favorite number. She tells us that the square of her favorite number equals her favorite number. Can we tell what her favorite number is? If not, what are the possible answers?

8. Ann decides to give us another hint at guessing her favorite number. She says that the square of her favorite number equals twice her favorite number. What is Ann's favorite number?

9. Jerry has a number in decimal notation which he converted to binary notation and got 1010. What is his number in decimal?

10. John has a number in binary notation which he converted to decimal notation and got 37. What is his number in binary?

11. Jack has a number in binary, but his last digit is missing! His number is 101*, where * is the missing digit.
    a. What are the two possibilities that his number can be in binary?

    b. Jill tells him his number is odd. What must Jack's number be?

12. Jack now has a different number in binary, again with a missing digit! His number is 10*1, where * is the missing digit.
    a. What are the two possibilities that his number can be in binary?

    b. Jill tells him that his number is divisible by 3. What must Jack's number be?

13. Now Jack has the number 11*1 in binary, again with a missing digit! Jill tells Jack that if he adds his number to the binary number 10, he gets the number 1111 in binary. What is Jack's number?

14. Now Jack has two numbers in binary with missing digits! Jill tells Jack if he adds his numbers, 11*1 and 1*01, he gets 11000 in binary. What are Jack's numbers?

15. On Halloween (October 31st), Martha goes trick-or-treating and gets a lot of candy! On November 1st, she gives half of her candy to one of her friends. On November 2nd, she gives half of her remaining candy to another one of her friends. She continues this on the 3rd, 4th, 5th, and 6th, but on November 7th she decides not to give any more candy away because she has only 2 pieces left! How many pieces of candy did Martha get on Halloween?

16. The mother of triplets, Annie, Benny, and Jenny, left some strawberries for them on the table. She asked them to divide the berries equally. First, Annie took a third. Later, Benny took a third of what was left. Finally, Jenny took 4 strawberries, which was a third of the remaining amount. How many strawberries did the mother leave for them?

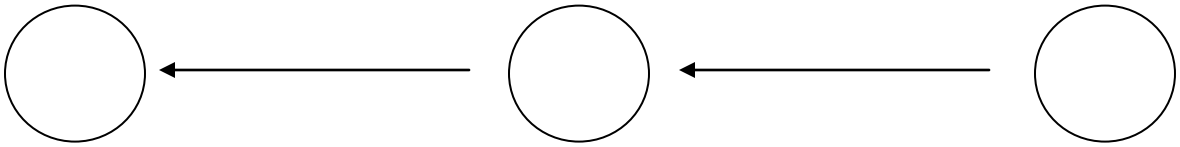
To solve this, fill in the correct numbers in the table below:

# of berries Jenny took	4
# of berries before Jenny came	
# of berries Benny took	
# of berries before Benny came	
# of berries Annie took	
# of berries before Annie came	

So, the number of berries the mother left is:

17. Barry, Larry, and Mary have a total of 60 books together. Larry has one more book than Barry, and Mary has one more book than Larry.
    a. How many books do they each own?

    b. Is this a backwards reasoning problem? If so, draw the operations and the numbers in the picture below.



18. Carrie bakes many pies on Sunday. On Monday, she gives half of her pies away, but bakes another half of a pie. She continues to give half of her pies away but bake another half of a pie on Tuesday and again on Wednesday. Finally, on Thursday, she gives half of her pies away, bakes another half of a pie, and ends up with two pies at the end of the day. How many pies did she bake on Sunday?

19. Jim has an amazing number trick. He says he can take any number, apply arithmetic operations to it, and always get the number 7! He says to do the following to any number you choose:
    i. Double the number
    ii. Add 5
    iii. Add 12
    iv. Subtract 3
    v. Divide by 2
    vi. Subtract your original number

    Try Jim's trick on a number of your choice! Do you get 7? Can you explain what is going on?