

Hotel Infinity

Junior Circle 10/11/2015

Imagine you are the owner of *Hotel Infinity*. This hotel is a very interesting place:

- There are infinitely many rooms in the hotel (numbered 1, 2, 3, 4, ...);
- Every room is on a different floor;
- Only one person can stay in each room at any moment;
- The guests are required to move to another room if asked to do so;

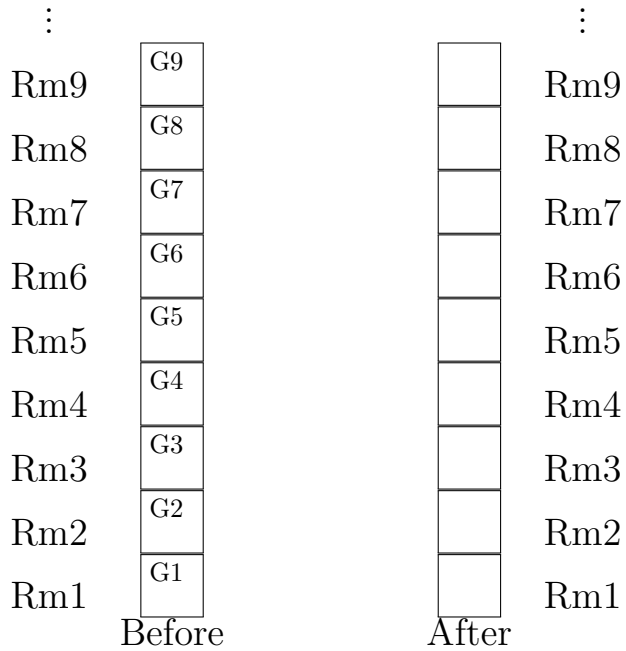
Here are some problems for you to solve:

1. In the morning, all the rooms in the hotel are taken. Then one more guest arrives.

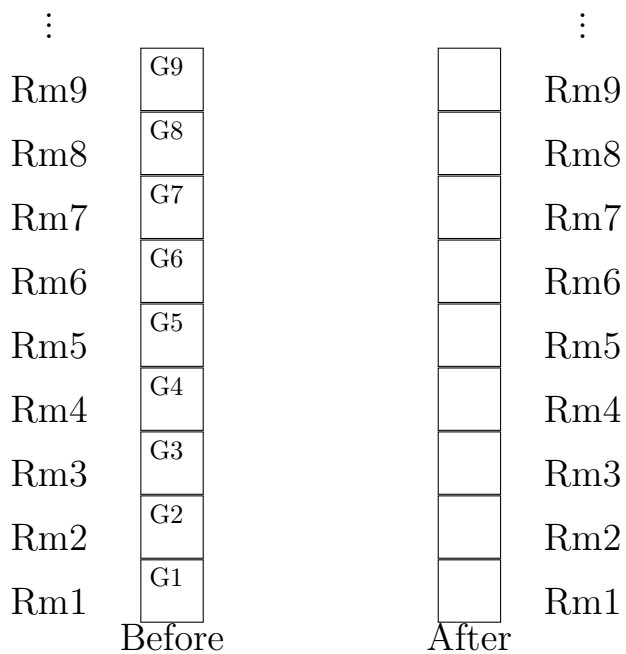
Can you accommodate the guest?

“Rm” represents the room number and “G” represents the guest.

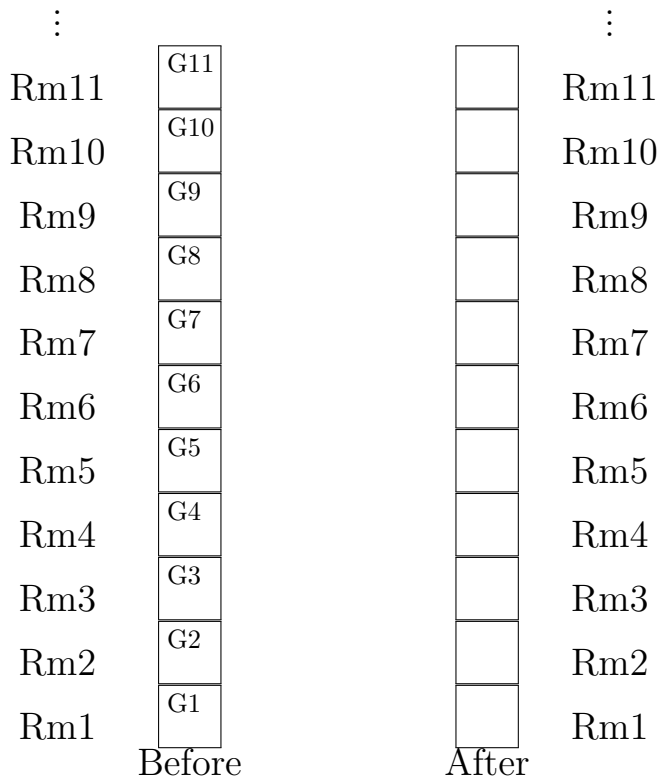
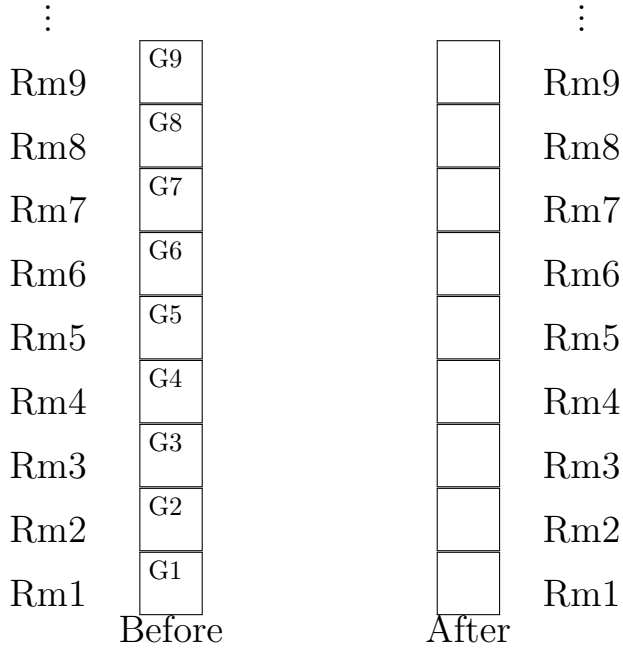
Use the hotel below to place the new guest(NG). (Hint: Try to put the guest into Room 1.)



2. Show a way to put the new guest(NG) into Room 6 and find a place for everyone else as well.



3. After this, things are quiet for a while. Then, 4 guests arrive. Can you find room for each of the new guests as well? Give two possible answers below to place the new guests(NG1, NG2, NG3, NG4).



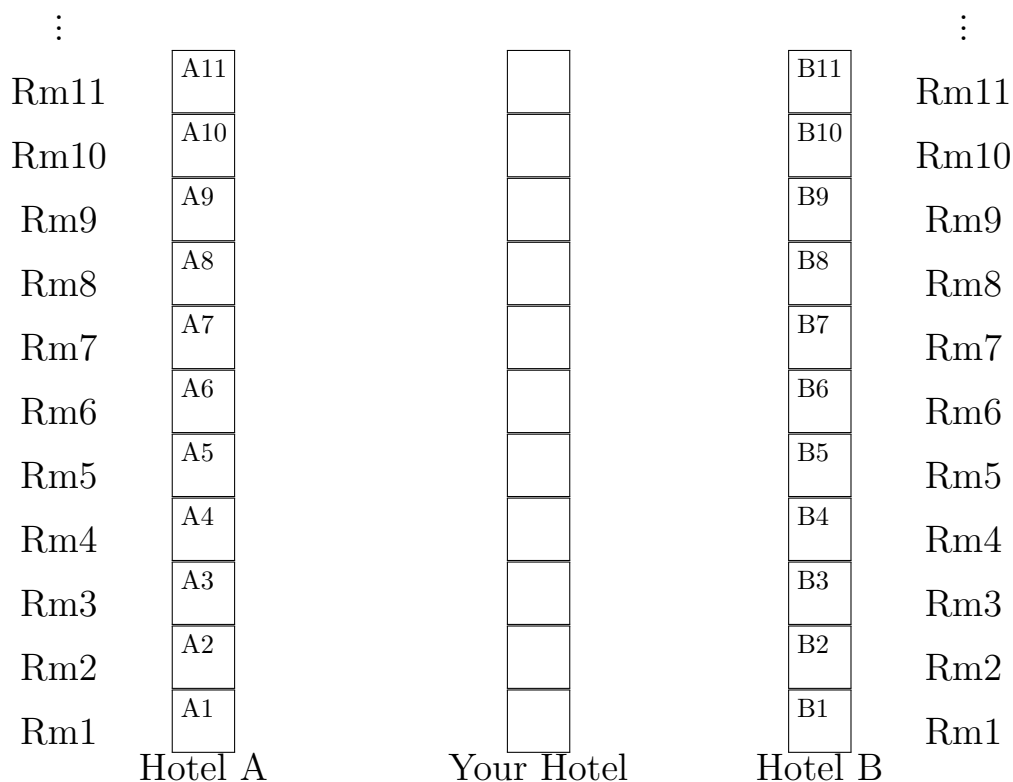
4. The four new guests insist on staying in rooms with prime numbers. Can this be done? Show a possible solution below:

⋮				⋮
Rm11	G11			Rm11
Rm10	G10			Rm10
Rm9	G9			Rm9
Rm8	G8			Rm8
Rm7	G7			Rm7
Rm6	G6			Rm6
Rm5	G5			Rm5
Rm4	G4			Rm4
Rm3	G3			Rm3
Rm2	G2			Rm2
Rm1	G1			Rm1
	Before		After	

5. Now imagine you have two siblings, each owning a hotel exactly the same as yours (with infinitely many rooms). However, they are located in different places. Both of your siblings' hotels are full, while yours is empty. All the guests from both of these other infinite hotels want to relocate to yours to be closer to the beach.

(a) Can you accommodate the guests from both of these hotels into your Hotel Infinity?

(b) If so, distribute the guests into your hotel.



(c) What other ways could you distribute the guests from the two hotels? Explain briefly.

6. Your friend in the nearest galaxy has his own Hotel *Double Infinity*. In his hotel, there are also infinitely many floors numbered $1, 2, 3, \dots$. However, each floor has 2 rooms so that

- rooms 1 and 2 are on floor 1;
- rooms 3 and 4 are on floor 2;
- rooms 5 and 6 are on floor 3;

(a) Which floor has room 23 and 24 on it?

(b) Which floor has rooms 2011 and 2012 on it?

(c) What rooms are on the 100th floor?

(d) What floor has rooms numbered $(2n - 1)$ and $2n$? (Hint: Look at the examples above.)

(e) Your friend says that his hotel has twice as many rooms as yours. Do you think he is right? Explain why or why not.