# Combinatorics

BEGINNERS 08/05/2015

Part IV: Permutations (Continued)

*Permutations* are arrangements that can be made by placing objects in a row. The order of the objects is important.

1. How many three digit numbers can you write using the digits 3, 3 and 4?

(a) Let's say that we have a 3, a **bold 3** and a 4. Write down all the numbers that you can make with the digits 3, **3** and 4.

(b) Considering the 3 and the bold **3** as two different digits, how many different numbers are there?

Even though they are colored differently, the 3 and the bold **3** have the same meaning:

3**3**4 is the same number as **3**34.
4**3**3 is the same number as 43**3**.
**3**43 is the same number as 34**3**.

For every permutation that we write, there will be another one in which the positions of the 3s are switched. And both numbers will obviously be the same!

Therefore, there are some *repetitions* among the numbers above.

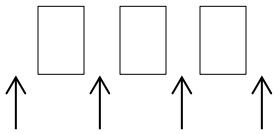
(c) Taking into account that 3 and bold **3** mean the same digit, how many different numbers written with digits 3, 3 and 4 are there?

Here is how we computed the number of arrangements of the digits 3, 3, 4:
First, pretend that the two 3s are different (one of them is bold). Then the number of arrangements is equal to 3! = 6. Then, take into account that the arrangements which are obtained from each other by switching the two 3s are actually the same. As a result, we get:

$$\frac{3!}{2!} = 3$$

2. How many four-digit numbers can you write using the digits 3, 3, 3, and 4?

   (a) Notice that three of the digits are equal to 3. This means that we just have to decide where to put the digit 4.



   This can be done in _____ ways.

   Thus, the total number of permutations is _____.

   (b) Now we will compute the number of rearrangements using the same method as in the first problem. Let's say that we have a 3, a bold **3**, an underlined <u>3</u>, and a 4. In how many ways can you rearrange 3, **3**, <u>3</u>, and 4?

   (c) Every two rearrangements which differ just by the order of 3s are the same. In how many ways can you rearrange 3, **3**, <u>3</u> in three slots?

   (d) How many distinct four-digit numbers written with 3, 3, 3, and 4 are there? Does your answer agree with the result of part (a)?

Word Bank

1. In how many ways can you rearrange the letters of the word IOWA?

2. In how many ways can you rearrange the letters of the word ARIZONA?
   (a) Pretending that the two letters A are different, how many permutations do you have?

   (b) Now taking into account that both As are identical, how many permutations do you have?

3. In how many ways can you rearrange the letters of the word COLORADO?

4. In how many ways can you rearrange the letters of word TENNESSEE?

Practice Problems

1. There are 39 cities in a country, each with one airport. Every pair of them is connected by an air route. How many air routes are there?

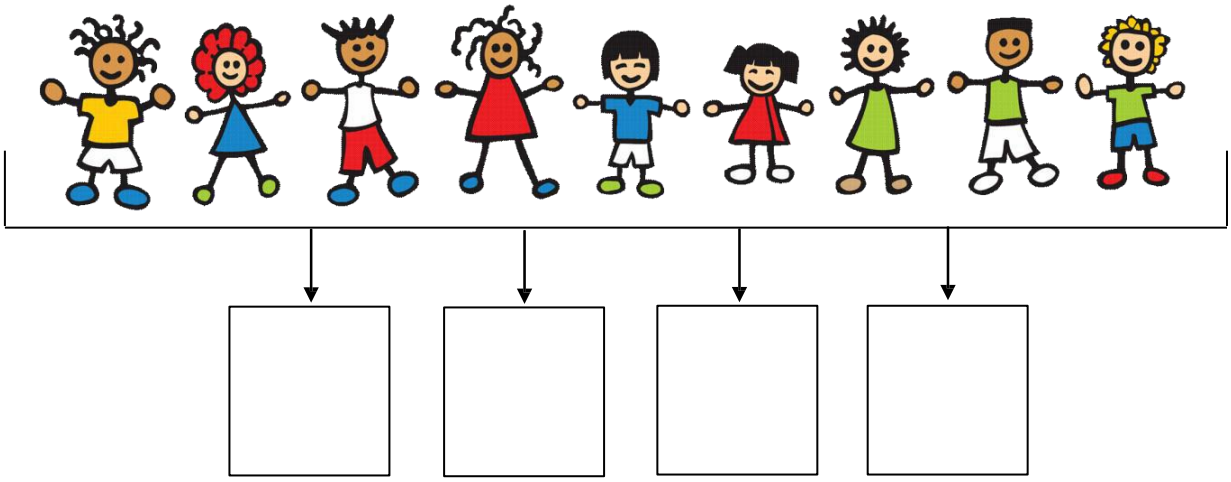
2. Josh's mother has two apples, three bananas, and four oranges. Every morning, for nine days, she gives one fruit to her son for breakfast. How many ways are there to do this?

3. How many "words" can be written using exactly five As and at most three Bs?

Part V: Combinations

Sometimes, the order in which you arrange things doesn't matter. For example, Ms. Cranberry has to choose four students from her class of nine to send for a mathematical contest. Does the order in which she picks the first, second, third and fourth student matter?

No, it does not!



i. How many different options does Mrs. Cranberry have for the first student? _____

ii. After selecting the first, how many options does she have for the second? _____

iii. Now, for the third? _____

iv. For the fourth? _____

We see that there are _____ ×_____ ×_____ ×_____ ways of choosing the students. Let us call this number A. However, this gives us the answer for the number of permutations (i.e., the order matters!)

Forget about the order now. If the names of the four students picked are Abe, Gus, Rob and Zed, it doesn't matter if the order is "Gus, Zed, Abe, Rob" or "Zed, Abe, Rob, Gus." So, in how many ways can you arrange these four students amongst themselves? ___ ×___ ×___ ×___ Let us call this number B. B is simply the number of ways in which you can rearrange four students. It is also the number of repetitions, right?

Similarly to what we did above when we had repetitions or *redundancies*, let's divide A by B. We find the number of all the permutations and divide by the number of all the *redundancies*.

What does $\frac{A}{B}$ show? Explain in your own words.

Challenge Yourself!

1. There are four kittens at the pet store. Your mom says that you may choose two to take home. In how many different ways can you choose two kittens from the litter of four?

2. The coach of the UCLA football team has to choose a captain and a deputy from his team of 10 students. How many ways are there to do that?

3. The coach of the USC football team, however, wants to choose a captain and a deputy as well as three assistants from his team of 10 students. In how many ways can he pick these four leaders?

4. Mr. Pi has to choose three girls and three boys to send for a debate. There are 14 girls and 11 boys in his class. In how many ways can he make the team?

5. On any given night, there can be between zero and four babysitters at home in the Simpsons' house. The babysitter company has eight employees from which it can choose to send babysitters to the Simpsons. How many possible combinations of babysitters can be at the house?