

Problems related to percents

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1. A percentage is a number expressed as a fraction of 100. One percent is defined as the following:

$$1\% = \frac{1}{100}$$

- (a) Write the following percentages as fractions. Simplify all of the fractions.

- 17%

$$\frac{17}{100}$$

- 60%

$$\frac{60}{100} = \frac{6}{10} = \frac{3}{5}$$

- 15%

$$\frac{15}{100} = \frac{3}{20}$$

- (b) Write the following fractions as percentages.

- $\frac{1}{4}$ 25%

- $\frac{2}{5}$ 40%

- $\frac{13}{50}$ 26%

- $\frac{23}{50}$ 46%

2. A pizza is divided into 12 equal slices. Johnny ate 75% of the pizza. How many slices of pizza did he eat?

$$\frac{75}{100} \times 12 = 9$$

\Rightarrow 9 pieces.

3. A giant pizza is divided into 20 equal slices. Clara ate 8 slices. What percentage of the pizza did she eat?

$$\frac{8}{20} \times 100 = 40\%$$

She ate 40% of the pizza.

4. After Ana ate 25% of a pizza, there were 12 slices left. How many slices were originally in the pizza?

75% of the pizza was left, which equals 12 slices.

$$\frac{75}{100} \cdot x = 12$$

$$\Rightarrow x = 12 \times \frac{100}{75}$$

$$= 12 \times \frac{4}{3} = 16$$

\therefore , there were 16 slices.

5. The price of an apple is $\frac{4}{5}$ of the price of a pear. Let a be the price of an apple, and p be the price of a pear.

(a) Relate a to p by completing the statement below.

$$a = \frac{4}{5} p$$

(b) What percentage of the price of a pear is the price of an apple?

80% [The pear is more expensive.]

(c) What fraction of the price of an apple is the price of a pear?

$$\frac{5}{4} a = p$$

so, $\frac{5}{4}$ of the price of an apple is the price of a pear.

(d) What percentage of the price of an apple is the price of a pear?

125%

[Remember, it is not 120%]

$$120\% = \frac{6}{5}, \text{ not } \frac{5}{4}$$

6. A gallon of milk is twice as expensive as a loaf of bread. Let m denote the price of a gallon of milk and b denote the price of a loaf of bread.

(a) Relate m to b .

$$m = 2b$$

[The gallon of milk is more expensive.]

(b) What percentage of the price of a gallon of milk is the price of a loaf of bread?

The loaf of bread is 50% the price of a gallon of milk.

(c) How many percents more is the price of milk compared to the price of bread?

The price of the gallon of milk is 100% more compared to the price of bread.

7. An eraser is 20% of the price of a pencil. Let e be the price of an eraser, and p be the price of a pencil.

(a) Relate e to p .

$$e = 20\% p$$

$$e = \frac{1}{5} p \quad \Rightarrow \quad 5e = p$$

(b) By how many percents is the pen more expensive than the pencil?

$$500\% \text{ of } e \text{ is } p.$$

Therefore, the pen is 400% more expensive.

8. If the price of eggs dropped by 60%, by what percentage would the price need to be raised in order to return to the original price?

Say, the price of the eggs was x .

After being dropped by 60%, the price is $x - 60\%x$
 $= 40\%x$.

To make it return to its original price,
it needs to be increased by 150%.

$$40\%x + 150\%(40\%x) = 40\%x + 60\%x = x$$

9. Amy put a certain amount of money into the bank. At the end of each year, the amount of money in the bank increases by 25%. After how many years will the amount of money she has be over twice the amount she had in the beginning?

Beginning x

$$\text{End of year 1: } x + 25\%x = x(1 + 25\%) = 1.25x$$

$$\begin{aligned} \text{End of year 2: } & 1.25x + 25\%(1.25x) \\ &= 1.25x + .3125x \\ &= 1.5625x \end{aligned}$$

[Try using fractions!]

$$\begin{aligned} \text{End of year 3: } & 1.5625x + 25\%(1.5625x) \\ &= 1.95x \end{aligned}$$

$$\text{End of year 4: } 1.95x + 25\%(1.95x) = 2.44x. \quad \text{So, after 4 years.}$$

10. Two bobcats are running a race. The length of the baby bobcat's jump is $\frac{1}{2}$ of the mother bobcat's jump. However, the baby bobcat makes $\frac{3}{2}$ as many jumps as the mother bobcat. Who is going to win the race?

(say) In time t , the mother bobcat covers 1 m.

So, in time t , the baby bobcat covers $1 \times \frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$ m.

So, the baby bobcat is slower.

Therefore, the mother bobcat wins.

11. The price of apples dropped so that you can now buy 25% more apples paying the same price as before. By what percentage did the price drop?

Let's say you could earlier buy 100 apples for \$100.

Now, you can buy 125 apples for \$100.

Earlier, the price of each apple was \$1.

Now, the price of each apple is $\frac{100}{125} = \$\frac{4}{5} = 80c$

So, the price dropped by 20%.

12. The humidity (the percentage of water) of mushrooms is 99%. The humidity of dried mushrooms is 98%. How many kilograms of dried mushrooms can you make starting from 100 kg of freshly picked mushrooms?

In 100 kg of freshly picked mushrooms, 99 kg is water.

For every 2 kg of pure mushrooms, there is 98 kg of water in dried mushrooms.

So, in every 1 kg of pure mushrooms, there should be 49 kg of water.

We can get 1 kg of pure mushrooms from the freshly picked mushrooms, with 49 kg of water, we have 50 kg of dried mushrooms.

13. The price of Halloween decorations dropped by a certain percentage on Halloween. The next day, the price dropped by the same percentage so that the final price was 25% of the original price. By what percentage did the price drop each time? (Hint: use fractions instead of percents.)

Initial price: x .

$$\text{Final price } \frac{25}{100} x = \frac{1}{4} x$$

Clearly $\frac{1}{2} \times [\frac{1}{2} x] = \frac{1}{4} x$.

So, the price dropped by 50% each time.

14. After the price of an adventure park ticket was discounted twice by 60%, the final price was \$36.

- (a) What was the price of the ticket before the discounts?

Original price: x

$$\text{Final price: } \$36 = 60\% (60\% x)$$

$$60\% \times 60\% \times x = 36$$

$$\Rightarrow x = \frac{36 \times 100}{64} \times \frac{100}{64}$$

$$= \$100$$

- (b) What was the total percentage discount (the discount needed to get from the initial price to the final price)?

Total percentage discount to
get from \$100 to \$36
is 64%.

15. John left 100 grams of a 1% salt solution on the table. After the water evaporated for some time, the remaining solution was a 2% salt solution. How much of the solution was left? (Note: A salt solution contains water and salt, and the salt does not vaporize.)

In 100 g of a 1% salt solution, there is 1g of salt and 99 g of water.

After some water evaporated, we were left with a 2% salt solution, out of which 1g is salt.

2% of 50 g is 1g.

So, 50 g of water evaporated, and we were left with 49 g of water and 1g of salt, giving a 2% salt solution.

16. The population of a town increased by 25% two years ago, and then dropped by 25% last year. If the population is now 4500 people, what was the population two years ago?

After a 25% drop, the population is 4500.

[x is the population 1 year ago.]

$$x - 25\%x = 4500$$

$$\text{So, } 75\%x = 4500$$

$$\text{So, } x = 4500 \times \frac{100}{75}$$

= 6000 was the population one year ago.

6000 was the result of a 25% increase.

[y is the population 2 years ago.]

$$y + 25\%y = 6000$$

$$125\%y = 6000$$

$$\Rightarrow \frac{5}{4}y = 6000$$

$$\Rightarrow y = 6000 \times \frac{4}{5}$$

$$\Rightarrow \underline{\underline{4800}}$$

\therefore 2 years ago, the population was 4800 people.

17. John bought a car for \$20,000. He sold the car for a 30% profit (meaning that the amount of money he sold the car for was 30% more than the amount he bought it). John then wanted to buy a car more expensive than the previous car he owned. He had to raise 30% more money in order to buy the new car. What was his overall profit or loss?

$$30\% \text{ of } 20,000 = 6000$$

The profit was \$6000.

So, he earned 26,000 on selling the car.

$$30\% \text{ of } 26,000 = \$7800$$

He bought the new car for \$33,800.

He earned \$6000, and paid \$33,800.

So, the loss was \$27,800.

18. A merchant bought an item at a certain cost. He tried to sell the item at a certain price but was not successful. After dropping the price by 20%, he was able to sell the item and make a 20% profit (when compared to the price that he paid for the item). If the merchant sold the item without the discount, what would be his profit?

Let's say that he bought the item for X dollars.

He tried to sell it for Y dollars.

$$\text{After dropping the price by } 20\% \rightarrow Y - 20\%Y = 80\%Y$$

$$\text{He made a } 20\% \text{ profit} \rightarrow X + 20\%X = 120\%X$$

$$\text{So, } 80\%Y = 120\%X$$

because it was sold at 120% the original price.

$$\Rightarrow Y = \frac{120\%}{80\%} X = \frac{3}{2} X = 150\%X$$

That would be a 50% profit.

Math Kangaroo Problems

19. There are five containers in a treasure chest. In each container there are three boxes and in each box there are 10 golden coins. The treasure chest, the containers, and the boxes are all locked. How many locks do you need to open to get 50 coins?

$$\begin{array}{l}
 \text{Treasure chest} \\
 \downarrow \\
 5 \text{ containers} \\
 \downarrow \\
 3 \text{ boxes each} \\
 \downarrow \\
 10 \text{ golden coins each}
 \end{array}
 \qquad
 \begin{array}{l}
 1 + 5 + 3(5) \\
 = 1 + 5 + 15 \\
 = 21 \text{ locks}
 \end{array}$$

20. On the right side of a certain two-digit number the same number has been written, creating a four-digit number. How many times greater is the new four-digit number than the original two-digit number?

Say, the two-digit number is XY .

So, the four-digit number is $XYXY$.

$$\text{So, } \frac{XYXY}{XY} = \frac{X(1000) + X(10) + Y(100) + Y}{XY}$$

$$= \frac{(1010)X + (101)Y}{XY}$$

$$= \frac{101(10X + Y)}{XY}$$

$$= \frac{101(XY)}{XY}$$

$$= 101 \text{ times}$$