

Beginner's Group

JULY 1, 2015

Warm Up: Logic Problem from The New York Times

Albert and Bernard just met Cheryl. "When is your birthday? Albert asked Cheryl.

Cheryl thought a second and said, "I'm not going to tell you, but I will give you some clues." She wrote down a list of 10 dates:

May 15, May 16, May 19

June 17, June 18

July 14, July 16

August 14, August 15, August 17

"My birthday is one of these," she said.

Then Cheryl whispered in Albert's ear the month, and only the month, of her birthday. To Bernard, she whispered the day, and only the day.

"Can you figure it out now?" she asked Albert.

Albert: I don't know when your birthday is, but I know Bernard doesn't know either.

Bernard: I didn't know originally, but now I do.

Albert: Well, now I know too!

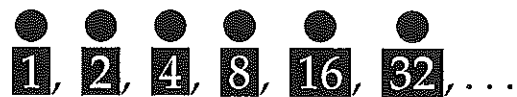
When is Cheryl's birthday?

July 16



Weighing with Powers of 2

In the country of Binary Land, the factory makes only the following weights (in kilograms):


1, 2, 4, 8, 16, 32, ...

You can buy any number of these standard weights from the factory and use them with a balance scale.

1. What is the pattern in the sequence of weights above?

These are powers of 2.

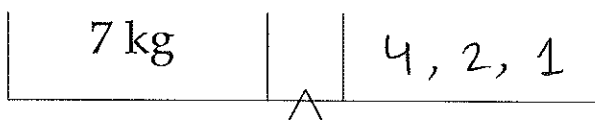
2. Write down the next 3 weights in the sequence:

64, 128, 256

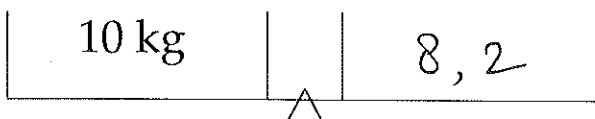
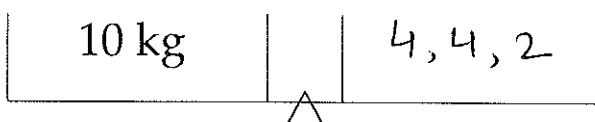
3. Balance each of the following objects using the weights

●
1, 2, 4, 8, 16,

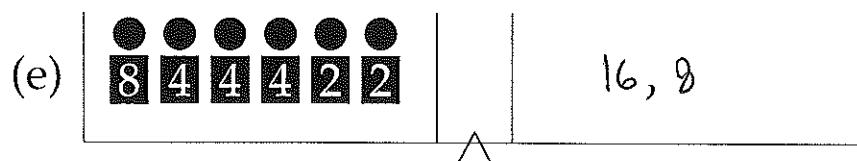
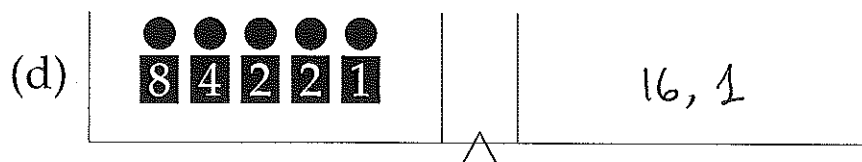
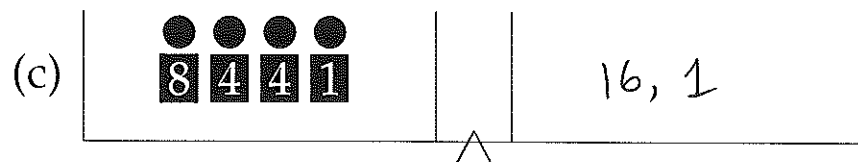
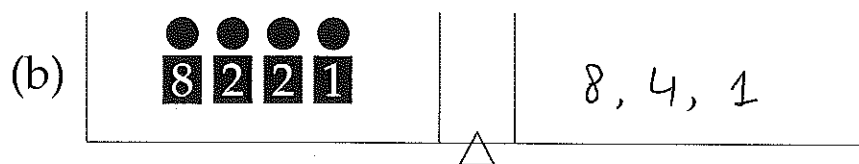
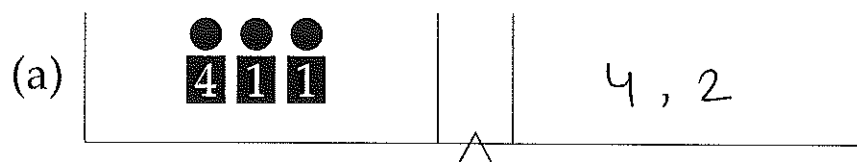
- (a) Find two different ways to balance a watermelon weighting 7 kilograms. (Note that you can use the same weight more than once).



- (b) Find two different ways to balance a metal ball weighing 10 kilograms. (Note that you can use the same weight more than once).



4. Now try to use each of the standard weight only once.
Can you balance the following:



5. Someone stole the 1 kilogram weight. What kind of weights can you balance now? (You still have all of the weights

$\overset{\bullet}{\square}2, \overset{\bullet}{\square}4, \overset{\bullet}{\square}8, \overset{\bullet}{\square}16, \dots$).

Only even weights, not odd.

6. The next day, the 2 kilogram weight got stolen, too. Which weights can you balance now? (You have the weights

$\overset{\bullet}{\square}4, \overset{\bullet}{\square}8, \overset{\bullet}{\square}16, \dots$).

Only multiples of 4.





7. They found the 1 kilogram weight! Which weights can you balance now? (You have the weights $\overset{\bullet}{\square}1, \overset{\bullet}{\square}4, \overset{\bullet}{\square}8, \overset{\bullet}{\square}16, \dots$).





All weights, because you can use 1 multiple times to get odd weights.

8. Balance the following. Do not use the same weight more than once. Fill in the table below:

- if you are using a certain weight, put 1 in its column;
- if you are not using a certain weight, put 0 in its column.

Some examples are filled in:

Weight	Write as sum of weights	 8	 4	 2	 1
1	$1 = \overset{\bullet}{\boxed{1}}$	0	0	0	1
2	$2 = \overset{\bullet}{\boxed{2}}$	0	0	1	0
3	$3 = \overset{\circ}{\boxed{2}} \overset{\circ}{\boxed{1}}$	0	0	1	1
4	$4 = \overset{\circ}{\boxed{4}}$	0	1	0	0
5	$5 = 4, 1$	0	1	0	1
6	$6 = \overset{\bullet}{\boxed{4}} \overset{\bullet}{\boxed{2}}$	0	1	1	0
7	$7 = 4, 2, 1$	0	1	1	1
8	$8 = 8$	1	0	0	0

Weight	Write as sum of weights				
9	$9 = \overset{\bullet}{\text{8}} \overset{\bullet}{\text{1}}$	1	0	0	1
10	$10 = 8, 2$	1	0	1	0
11	$11 = 8, 2, 1$	1	0	1	1
12	$12 = 8, 4$	1	1	0	0
13	$13 = \overset{\bullet}{\text{8}} \overset{\bullet}{\text{4}} \overset{\bullet}{\text{1}}$	1	1	0	1
14	$14 = 8, 4, 2$	1	1	1	0
15	$15 = 8, 4, 2, 1$	1	1	1	1

9. Major robbery!

- (a) After many weights were stolen you are left only with the weights $\overset{\bullet}{1}$, $\overset{\bullet}{2}$, $\overset{\bullet}{4}$ and $\overset{\bullet}{8}$. Can you balance 18 kilograms using each weight no more than once? Why or why not? What weights can you balance?

The maximum I can balance with these weights, using each only once, is 15 kg.

I can only balance anything equal to or lighter than 15 kg.

So, I cannot balance 18 kg.

- (b) What is the largest number you can write as a sum of some the numbers 1, 2, 4, 8 without using the same number more than once?

$$8 + 4 + 2 + 1 = 15 \text{ kg}$$

(c) Can you balance 30 kilograms with the given weights

1, 2, 4, 8, 16, ...

without using any weight more than once. That is, write 30 as a sum of several of these numbers, without repeating any of the numbers:

$$30 = 16 + 8 + 4 + 2.$$

Now write down the string of 0s and 1s that corresponds to 30. (Write 1 under the numbers that are used in the sum above. Write 0 under the numbers that are not used in the sum above):

	16	8	4	2	1
30	1	1	1	1	0

(d) Can you write down 57 as sum of some of the numbers

1, 2, 4, 8, 16, 32, ...?

Do not use the same number twice.

$$57 = 32 + 16 + 8 + 1.$$

Now write down the string of 0s and 1s that corresponds to 57:

	32	16	8	4	2	1
57	1	1	1	0	0	1

10. What number corresponds to the following strings of 0s and 1s. Fill in the table:

32	16	8	4	2	1	computation	number
0	1	1	0	1	0	$16 + 8 + 2 = 26$	26
0	1	1	0	1	0	$16 + 8 + 2 = 26$	26
0	0	1	1	0	1	$8 + 4 + 1 = 13$	13
1	1	0	0	1	0	$32 + 16 + 2 = 50$	50
1	1	1	1	1	1	$32 + 16 + 8 + 4 + 2 + 1 = 63$	63

11. Every number can be written in *binary notation*:

$$1 = \boxed{1}, \quad 2 = \boxed{1}\boxed{0}, \quad 3 = \boxed{1}\boxed{1}, \quad 4 = \boxed{1}\boxed{0}\boxed{0}.$$

Find the missing numbers or binary notation:

$$\begin{array}{lll} \boxed{1}\boxed{0}\boxed{1} = 5 & \boxed{1}\boxed{1}\boxed{0} = 6 & \boxed{1}\boxed{0}\boxed{0}\boxed{1} = 9 \\ \boxed{1}\boxed{1}\boxed{0} = 6 & \boxed{1}\boxed{1}\boxed{1} = 7 & \boxed{1}\boxed{0}\boxed{0}\boxed{0} = 8 \end{array}$$

Here the numbers where each digit is written inside of a square are written in binary notation.

mm

11101

110001

1000

Binary Notation II

Decimal Notation:

1. What is the value of digit 8 in the number 938?

$$8$$

2. What is the value of digit 8 in the number 581?

$$8 \times 10 = 80$$

3. What is the value of digit 8 in the number 824?

$$8 \times 100 = 800$$

4. Let's look at the number 3456

- (a) What is the value of the digit 3?

$$3 \times 1000 = 3000$$

- (b) What is the value of the digit 5?

$$5 \times 10 = 50$$

Binary Notation:

1. Put the numbers 7, 15, 20, 29, 36, 43 and 50 into binary notation by filling out the table below:

Number	32	16	8	4	2	1
7				1	1	1
15			1	1	1	1
20		1	0	1	0	0
29		1	1	1	0	1
36	1	0	0	1	0	0
43	1	0	1	0	1	1
50	1	1	0	0	1	0

- (a) What is the value of the rightmost digit 1 in the binary notation of the number 7?

1

- (b) What is the value of the rightmost digit 1 in the binary notation of the number 20?

4

- (c) What is the value of the rightmost digit 1 in the binary notation of the number 36?

4

2. Write the following numbers in binary notation:

$$(a) 5 = 101$$

$$(b) 9 = 1001$$

$$(c) 17 = 10001$$

$$(d) 28 = 11100$$

$$(e) 63 = 111111$$

3. Convert the following numbers from binary notation to decimal notation:

$$(a) \boxed{1}\boxed{1} = 3 \quad [1 \times 1 + 1 \times 2 = 3]$$

$$(b) \boxed{1}\boxed{0}\boxed{0}\boxed{0}\boxed{1} = 17 \quad [1 \times 1 + 1 \times 16 = 17]$$

$$(c) \boxed{1}\boxed{0}\boxed{1}\boxed{1}\boxed{1} = 23 \quad [1 \times 1 + 1 \times 2 + 1 \times 4 + 1 \times 16 = 23]$$

$$(d) \boxed{1}\boxed{1}\boxed{0}\boxed{1}\boxed{1} = 27 \quad [1 \times 1 + 1 \times 2 + 1 \times 8 + 1 \times 16 = 27]$$

$$(e) \boxed{1}\boxed{1}\boxed{0}\boxed{1}\boxed{0}\boxed{1}\boxed{0} = 106 \quad [1 \times 2 + 1 \times 8 + 1 \times 32 + 1 \times 64 = 106]$$

4. Solve the following addition problems. [Hint: Carrying may be involved!] Check your work by solving the problem in decimal notation on the side.

$$\begin{array}{r}
 \boxed{1} \boxed{0} \boxed{1} \boxed{0} \\
 + \quad \boxed{1} \boxed{0} \boxed{1} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 10 \\
 + 5 \\
 \hline
 15
 \end{array}$$

$$\begin{array}{r}
 \boxed{1} \boxed{0} \boxed{1} \boxed{1} \\
 + \quad \boxed{1} \boxed{0} \boxed{0} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 11 \\
 + 4 \\
 \hline
 15
 \end{array}$$

$$\begin{array}{r}
 \\
 \boxed{1} \boxed{0} \boxed{1} \boxed{1} \\
 + \boxed{1} \boxed{0} \boxed{1} \boxed{0} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 11 \\
 + 10 \\
 \hline
 21
 \end{array}$$

$$\begin{array}{r}
 \\
 \boxed{1} \boxed{1} \boxed{1} \boxed{1} \\
 + \boxed{1} \boxed{1} \boxed{1} \boxed{0} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 15 \\
 + 14 \\
 \hline
 29
 \end{array}$$

5. Solve the following subtraction problems. Check your work by solving the problem in decimal notation on the side.

$$\begin{array}{r} \boxed{1} \boxed{1} \boxed{1} \boxed{1} \\ - \boxed{1} \boxed{0} \boxed{0} \boxed{1} \\ \hline 0110 \end{array}$$

$$\begin{array}{r} 15 \\ - 9 \\ \hline 6 \end{array}$$

$$\begin{array}{r} \boxed{1} \boxed{0} \boxed{1} \boxed{1} \\ - \quad \quad \boxed{1} \boxed{1} \\ \hline 1000 \end{array}$$

$$\begin{array}{r} 11 \\ - 3 \\ \hline 8 \end{array}$$

$$\begin{array}{r} \boxed{1} \boxed{1} \boxed{0} \boxed{1} \\ - \quad \quad \boxed{1} \boxed{1} \\ \hline 1010 \end{array}$$

$$\begin{array}{r} 13 \\ - 3 \\ \hline 10 \end{array}$$

$$\begin{array}{r} \boxed{1} \boxed{0} \boxed{0} \boxed{1} \\ - \quad \boxed{1} \boxed{1} \boxed{0} \\ \hline 0011 \end{array}$$

$$\begin{array}{r} 9 \\ - 6 \\ \hline 3 \end{array}$$

6. Compute the sum $1 + 1 + 2 + 4 + 8 + 16 + 32$ like this: first, convert all numbers to binary. Then, carry out the addition. Finally, convert your result to a decimal number.

				1	1
			1	1	0
		1	1	0	0
	1	1	1	0	0
1	1	0	0	0	0
1	0	0	0	0	0
0	0	0	0	0	0

+ 1 = 64

7. Solve the following addition and subtraction problems. Check your results by solving the problem in decimal notation on the side.

$$\begin{array}{r}
 \begin{array}{ccccc}
 \boxed{1} & \boxed{0} & \boxed{0} & \boxed{0} & \boxed{1} \\
 & \boxed{1} & \boxed{0} & \boxed{0} & \boxed{1} \\
 & & \boxed{1} & \boxed{0} & \boxed{1} \\
 + & & & & \\
 \hline
 & 1 & 1 & 1 & 1
 \end{array}
 \end{array}$$

Diagram illustrating the addition of three numbers (10101, 1010, and 1) to find their sum (32). The numbers are represented by boxes containing digits, and the sum is shown below a horizontal line.

Handwritten calculation on the right:

$$\begin{array}{r} 21 \\ + 10 \\ + 1 \\ \hline 32 \end{array}$$

$$\begin{array}{r} 12 \\ + 6 \\ + 7 \\ \hline 25 \end{array}$$

$$\begin{array}{r} 31 \\ + 15 \\ + 3 \\ \hline 49 \end{array}$$

$$\begin{array}{r} 24 \\ -9 \\ \hline 15 \end{array}$$

$$\begin{array}{r} 21 \\ -9 \\ \hline 12 \end{array}$$