Problems from a Moscow Math Olympiad

October 9, 2014

The following set of problems was given to 5th graders on a Moscow Math Olympiad:

1. The weight Winnie the Pooh holding 5 jars of honey is 25 kg. The weight of Winnie the Pooh holding 3 jars of honey is 19 kg. How much does Winnie the Pooh weight?

   The weight of 2 jars is 6 kg.
   $\Rightarrow$ weight of 1 jar = 3 kg.
   $\Rightarrow$ weight of 5 jars = 15 kg.
   $\Rightarrow$ weight of Winnie the Pooh = 25-15
   = 10 kg
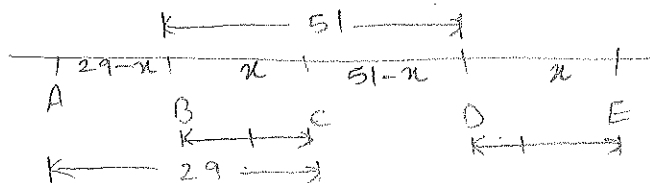
2. Cross out 3 digits in the number

   $$2\,0\,1\,2\,2\,0\,1\,2\,2\,0\,1\,2 \qquad 1\,2\,0\,1\,2\,2\,0\,1\,2$$

   to get the smallest possible nine digit number.

3. Points A, B, C, D, E lie on a line (in that order). Given that $AC = 29$cm, $BD = 51$cm, $BC = DE$, find the distance AE.
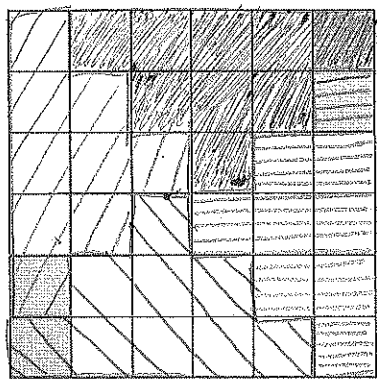

$$AE = AB + BC + CD + DE$$
$$= (29-x) + x + (51-x) + x$$
$$= 29 + 51$$
$$= 80 \text{ cm}$$

4. The first several sheets of an old book are missing. The remaining pages are numbered from 7 to 26. How many sheets from the book are remaining?
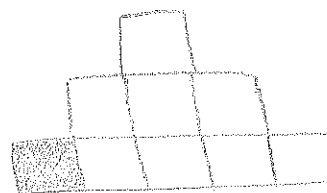
From page 7 to page 26, there are 20 pages.
So, 10 sheets.

5. Cut the square of size 6 × 6 into 4 parts equal in size and shape in such a way each part contains exactly one shaded square. (See the picture below)

All four pieces look like this:

6. Maria multiplied a two digit number by itself. The result is a four digit number in which the first two digits are equal to each other and the last two digits are equal to each other. What number did Maria start with?

(a) To help solve this problem, we can expand the four digit number by using the following relationship:

$$XXYY = XX \times 100 + YY$$

where X and Y are digits. Use this relationship to show that the number $XXYY$ must be divisible by 11.

$XXYY = XX \times 100 + YY$
$= 11 \times X \times 100 + 11 \times Y$
$= 11 \times (100X + Y)$
So, $XXYY$ is divisible by 11.

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(b) What does this tell you about the two digit number Maria started with?

Since XXYY is the square of a number, 11 must appear twice in its factorization.

So, the original two-digit number is also divisible by 11.

(c) Can you now determine which number she started with?

Multiples of 11 squared:

$11^2 = 121$
$22^2 = 484$
$33^2 = 1089$
$44^2 = 1936$
$55^2 = 3025$
$66^2 = 4356$
$77^2 = 5929$
$88^2 = 7744$

So, the original number is 88.

7. An astrologist predicted that:

(a) If today is a rainy, then tomorrow is sunny;

(b) If today is snowy, then tomorrow is rainy;

(c) If today is cold, then tomorrow is windy;

(d) If today is sunny, then tomorrow is warm;

(e) If today is warm, then tomorrow is cold;

(f) If today is cold, then tomorrow is cloudy;

(g) If today is windy, then tomorrow is snowy,

(h) If today is cloudy, then tomorrow is rainy.

Miraculously, all of the predictions were true last January. Given that January 1st was windy and sunny, describe the weather on January 5th.

Jan 1: Windy, sunny

Jan 2: Snowy, warm

Jan 3: Rainy, Cold

Jan 4: Sunny, Windy, Cloudy

Jan 5: Warm, Snowy, Rainy

8. Four inhabitants of the *Knights and Liars* island were asked: "How many of you are knights?" They gave 4 different answers in response. How many knight could have been there? (Consider all possible scenarios. Rememer that Knights always tell the truth and Liars always lie). [Their answers are restricted to 1-4]

If they all gave different answers,
the four answers are 1, 2, 3 and 4.

So, we cannot have 2, 3 or 4 knights because all answers are different and 2 or more could not be saying the truth.

So, 1 is a possible answer.
Also, we might have 0 knights if they are all lying.

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So, 0 or 1 knight.

9. Magic Elms grow to be 15 m tall, and Magic Birches grow to be 10 m tall. It takes 9 hours for a Magic Elm to grow to full height. It takes 2 hrs for a Magic Birch to grow to full height. If you plant a Magic Elm at noon and Magic Birch at 2pm, at what time will they be of the same height?

Magic Elms: 15m in 9 hours $\Rightarrow$ 15/9 m in 1 hour
Magic Birches: 10m in 2 hours $\Rightarrow$ 5 m in 1 hr.

In x hours, a Magic Elm would be $\frac{15x}{9}$ m tall.

In $(x-2)$ hours, a Magic Birch would be $5(x-2)$ m tall.

So, $\frac{15x}{9} = 5(x-2)$

$\Rightarrow 3x = 9(x-2) \Rightarrow 18 = 6x \Rightarrow x = 3$ hours.

At 3 o'clock, both would be 5m.

10. A city is divided into 4 districts. Each district has a boundary of 40 km. Districts are separated from each other by roads. The total distance of these roads is 30 km. There is also a road going around the city. What is the length of this road? Use the picture below to help you solve the problem. (Note that the boundary of each of the districts consists of a part of the road going around the city and roads separating it from other districts).
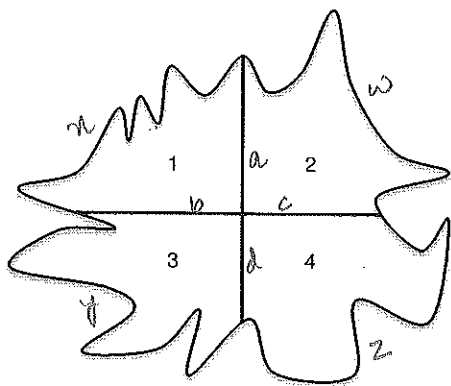
$a + b + c + d = 30$ km

$w + a + c = 40$
$x + b + a = 40$
$y + b + d = 40$
$z + c + d = 40$

w: curved road around district 1
Likewise, x, y, z : curved roads around districts 2, 3, 4.

Adding all, $w + x + y + z = 160$
$+ 2a + 2b + 2c + 2d$

So, $w + x + y + z = 160 - 60$
$= 100$ km

OR.
Adding up the boundaries of all four districts, we get
$4 \times 40 = 160$ km.
Subtracting the inner roads (each counted twice) $= -60$
So, $160 - 60 = 100$ km.