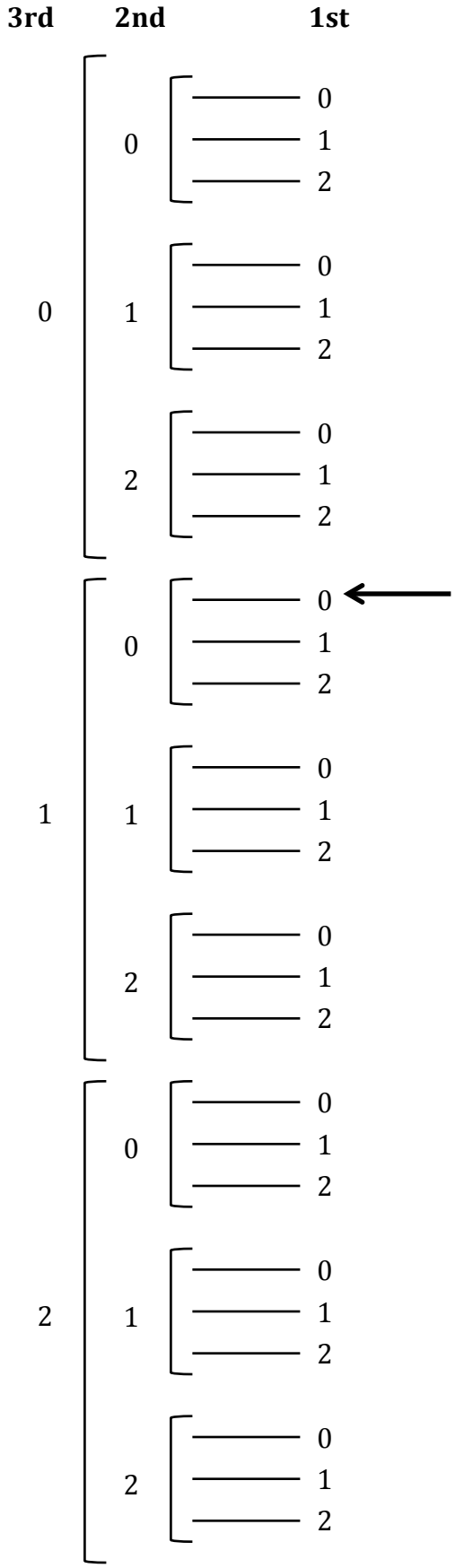
Card Trick with Number Bases

May 31, 2015

Impress your friends using Martin Gardner's card trick, which utilizes math in number base 3! The following steps explain the trick without explaining the mathematics behind it.

- The magician picks out 27 distinct cards from a standard card deck.
- The magician asks the volunteer to pick a random card and memorize it without showing the chard to the magician. Assume in this case, the card is the king of hearts. This card is then put back into the deck.
- The magician will then ask the volunteer for his/her favorite number. In this scenario, assume the volunteer's favorite number is 10.
- The cards are then dealt into three piles, where the top card is placed in the left pile, the next card in the middle, the next in the right, and the process is repeated until all cards are used.
- The volunteer points to which pile his/her card is in.
- The magician puts each pile together in a specific order, depending on the favorite number. The process of dealing the cards, pointing to the right pile, and rearranging the piles is repeated 2 more times.
- Then, the magician will count out the cards, starting from the top, until the tenth card is put down. The tenth card is flipped, and it is the king of hearts!

However, if you want to be the magician and impress your friends, you have to understand how the trick works. Let's assume that the three piles that are laid out are labeled 0, 1, and 2, where 0 represents the top pile, 1 is the middle pile, and 2 is the bottom pile. Now, we can try to understand the card trick analyzing the diagram and reading the explanations on the following page.



- The horizontal lines represent the final placement of the cards. See that the arrow points to the tenth card. The left column shows the three piles after the 3rd time the piles were picked up (the final round). The top pile is labeled "0", the middle "1", and the bottom "2".
- The top three cards from the 3rd top pile came from the previous top pile. The next three cards came from the previous middle pile, and last three cards came from the previous bottom pile. Similarly, we can label the 2nd column by "0", "1", and "2", shown on the left. Think about how this works!
- Also, within each group of three cards, the top one came from the top pile in the 1st round, the middle came from the middle pile of the 1st round, and the bottom came from the bottom of the 1st round. Thus, we finish our labeling in the right column.
- Therefore, if we want the card to be in the 10th position, we have to put the pile containing that card on the top during the first round. Then we put the pile with the card of interest on the top for the second round. Finally, the pile must be put in the middle on the last round. **If we put the pile with the card of interest on the top in the final round instead of the middle, what position would the card be in?**

- Recall that when we write a number *n* in base 3, we can write it as:

$$n_{10} = \overline{abc_3} = a\times 9 + b\times 3 + c\times 1$$

Note that the 3rd stage changes the position of a card by 9, the middle by 3, and the right by 1. Therefore, we can use base 3 arithmetic to determine how to position the piles. Since the top card is placed in piles 000, instead of 001, the placement of piles with favorite number *n* will be based on *n-1* written in base 3.

1. To further understand how the trick works, it is beneficial to trace out the position of the cards. We will be using the same scenario where the favorite number is 10. Below is a diagram showing the three piles at each stage. The card with the "X" on it is the card of choice. The cards need to be rearranged so that it ends up being the tenth card.

   (a) For the first stage, determine which piles should be the top, middle, and bottom. Then color each card in the top pile blue, the middle pile red, and the bottom pile green.

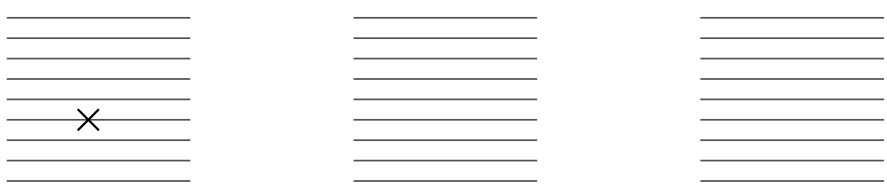
   (b) Color each card in the second stage to track the movement of each card. Note that each card should keep the same color (but the positions of the colors change!)

   (c) In the second stage, write an "X" on the card of interested. Then label which pile should be the top, middle, and bottom.

   (d) Repeat this process for the third stage: color the cards, denote the card of interest with an "X", and label which pile will be the top, middle, and bottom pile.

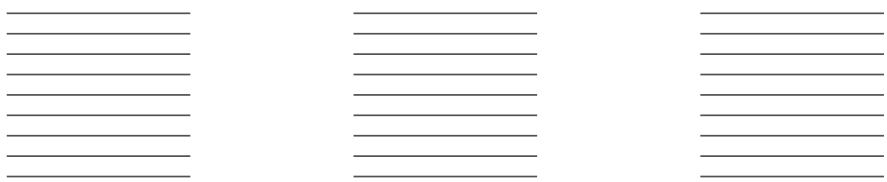
   (e) Which position is the card of interest in?

Stage 1



Stage 2

Stage 3



2. Determine how to position the three piles in each round by filling out the table below for each favorite number.

Fav num	Num of cards on top	Num of cards on top in base 3	Position of three piles
10	9	100	T T M
2			
11			
15			
26			
19			

- Practice the trick using 27 cards and using 19 as the favorite number.

3. Come up with the trick for base 2 where 8 cards are used by drawing the two corresponding diagrams (similar to the diagrams on pages 2 and 3) .

- How big is each pile?

- How many piles do you have in each round?

- How many rounds are there?

- How would you position the piles if the volunteer states that his/her favorite number is 5? Test your answer by using 8 cards.

4. Why do you think magicians generally use base 3 instead of base 2 for this card trick?

5. For each base (2, 3, 4), fill in the following table:

base	Number of cards	Number of rounds
3	27	3
3	9	
3		4
2	4	
2		
2		
4		
4		
4		