Vectors

May 17, 2015

Vectors are mathematical quantities that have direction and magnitude, and can be pictured as arrows. This is in contrast to *scalars*, which are quantities that have a numerical value but no direction.

1. Identify the following quantities as either vectors or scalars.

   - Mass

     Scalar

   - Velocity

     Vector

   - Temperature

     Scalar

   - Distance

     Scalar

   - Density

     Scalar

   - Force

     Vector

   - Acceleration

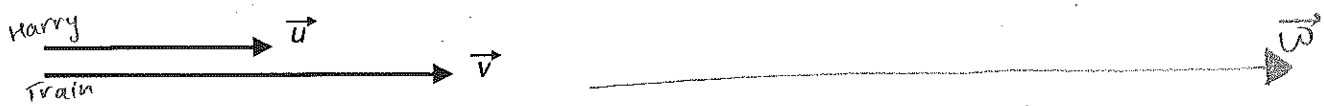
     Vector

   - Volume

     Scalar

Adding vectors on a line

2. Harry Potter rides on a train that is moving with a velocity $\vec{v}$. Harry moves with a velocity $\vec{u}$ with respect to the train. Harry moves in the same direction that the train moves, and the vectors for Harry's and the train's movements are shown below. Harry then casts the invisibility spell on the train. Draw the vector for Harry's movement as seen by a viewer standing outside of the train. Denote this vector by $\vec{w}$. (*Hint:* think about how large the vector needs to be, and which direction it needs to go in).

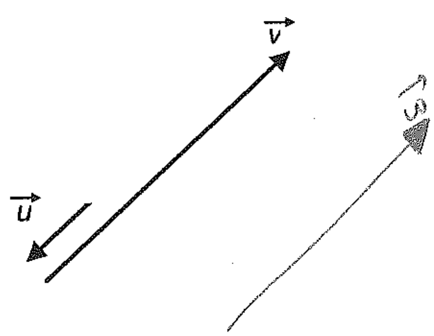


$\vec{w}$ is in the direction of $\vec{v}$ and $\vec{u}$.
It is the sum of the length of $\vec{u}$ and $\vec{v}$.

3. The river flows with a velocity $\vec{u}$. A boat moves on the river with a velocity $\vec{v}$ with respect to the river (in the direction of the river). The two vectors are shown below. Draw the vector that shows the boat's velocity with respect to someone standing on the shore of the river.

Similarly, someone on the shore would see the boat to move as fast as the combined speed of the river and the boat.

4. A passenger in a plane walks from the front to the back of the plane with a velocity $\vec{u}$, while the plane flies in one direction with a velocity $\vec{v}$. Draw the vector showing the passenger's velocity with respect to an outside observer.

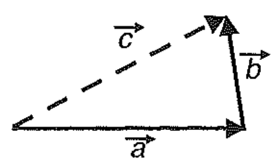


Effectively, the passenger is moving in the direction in which the plane is flying because the plane is moving much faster than the person.

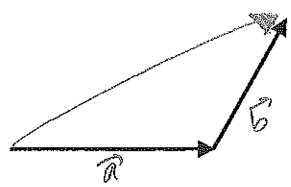
However, since he is walking in the opposite direction, his speed is not exactly equal to (it is less than) that of the plane.

Adding vectors on the plane

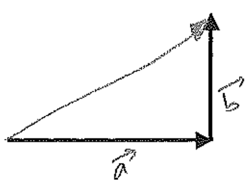
When talking about vectors, it is common to refer to the end with the arrow as the "head", and the starting point as the "tail". When adding vectors $\vec{a}$ and $\vec{b}$, the first step is to move the vectors in such a way that the head of $\vec{a}$ touches the tail of $\vec{b}$. Then, you connect the tail of $\vec{a}$ to the head of $\vec{b}$. Notice that this is what you did when adding vectors on a line. Below is an example showing the addition of vectors $\vec{a}$ and $\vec{b}$, resulting in vector $\vec{c}$. This is often referred to as "head-to-tail addition" method because you are putting the head of the second vector ($\vec{b}$) to the tail of the first vector ($\vec{a}$).



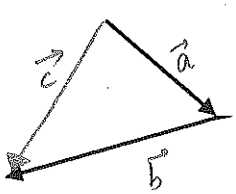
5. Add up the following vectors.



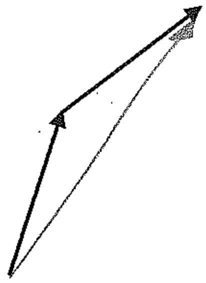
(a)



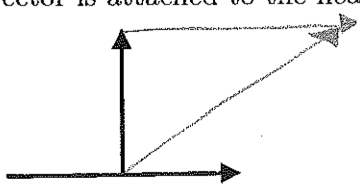
(b)



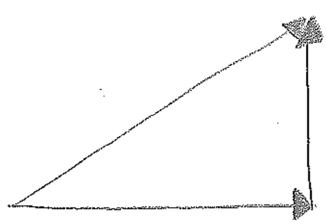
(c)

(d) 

6. In the example below, redraw the vectors in such a way that the tail of the horizontal vector is attached to the head of the vertical one. Then add the vectors.

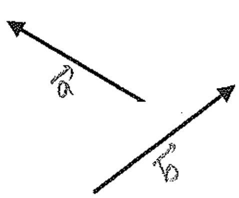


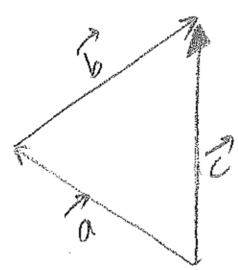
- Now repeat the problem by drawing the tail of the vertical vector attached to the head of the horizontal one. Is your answer different?

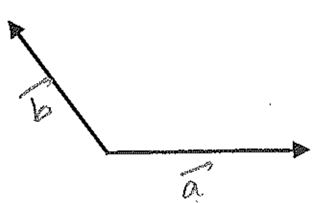


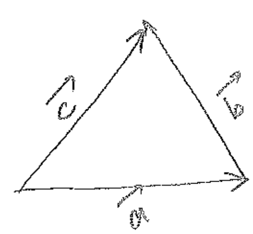
No.

7. Add up the following vectors.

(a) 



(b) 



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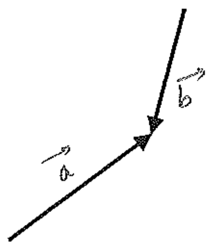
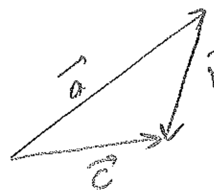
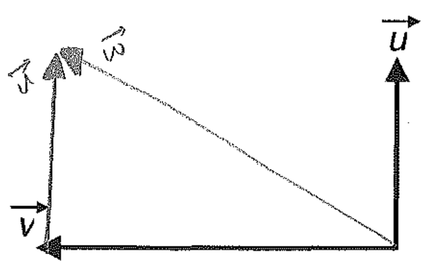


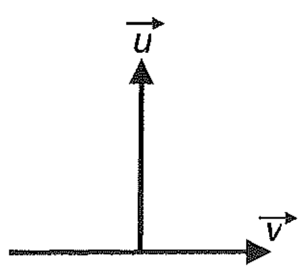


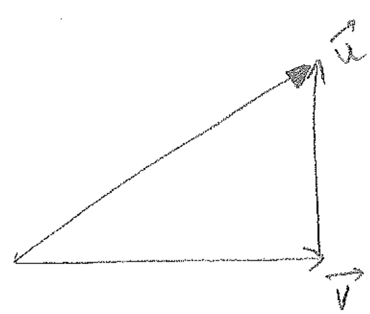
8. You are walking with a velocity $\vec{u}$ on a boat which is moving with a velocity $\vec{v}$, as shown below. Draw the vector showing your velocity with respect to an outside viewer standing on the shore.



$\vec{u}+\vec{v}=\vec{w}$

9. The train Harry rides on is a steam train, and a lot of steam is emitted to the atmosphere. The steam leaves upward with a velocity $\vec{u}$ while the train moves horizontally with a velocity $\vec{v}$. Draw the vector that shows the velocity of the steam with respect to an outside viewer.





$\vec{w}=\vec{u}+\vec{v}$

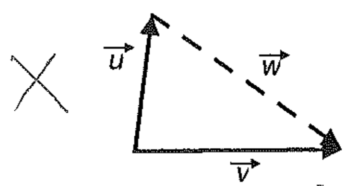
Subtracting vectors on the plane

Recall that subtraction is the operation opposite to addition.

- For numbers, performing subtraction $a - b$ means finding such a number c that $b + c = a$. For example

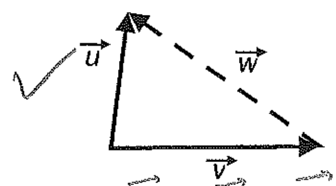
$$5 - 3 = 2 \qquad \text{because} \qquad 3 + 2 = 5.$$

- Similarly, finding $\vec{u} - \vec{v}$ means finding a vector $\vec{w}$ such that $\vec{u} = \vec{v} + \vec{w}$. Below are two drawings, one where $\vec{w} = \vec{u} - \vec{v}$, and one where $\vec{w} \neq \vec{u} - \vec{v}$. Circle the one where $\vec{w} = \vec{u} - \vec{v}$.



$\vec{V} = \vec{u} + \vec{w}$

$\Rightarrow \vec{w} = \vec{V} - \vec{u}$



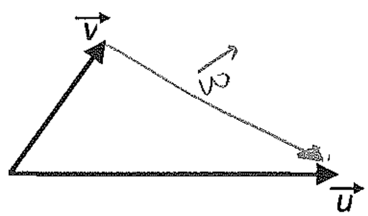
$\vec{u} = \vec{V} + \vec{w}$

$\Rightarrow \vec{w} = \vec{u} - \vec{V}$

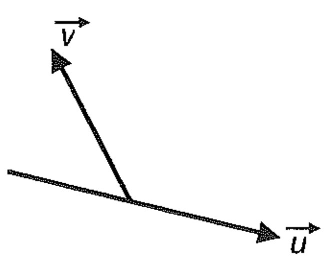
10. Subtract the vectors shown below. That is, draw the vector equal to $\vec{u} - \vec{v}$.

$\vec{V} + \vec{w} = \vec{u}$

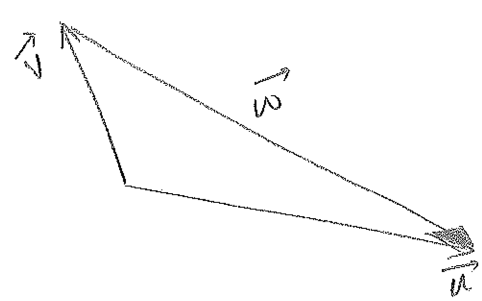
So, $\vec{w} = \vec{u} - \vec{V}$



(a)



(b)



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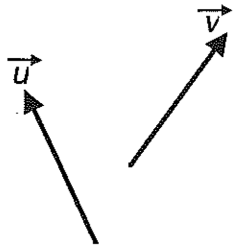
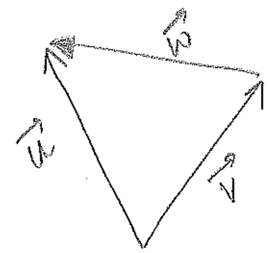
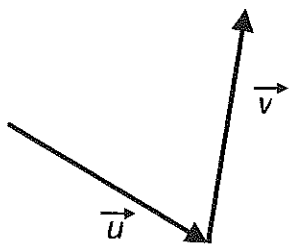


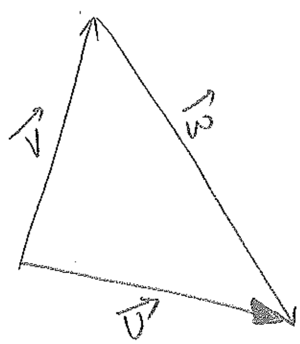


$\vec{v}+\vec{w}=\vec{u}$

$\vec{w}=\vec{u}-\vec{v}$

(d)



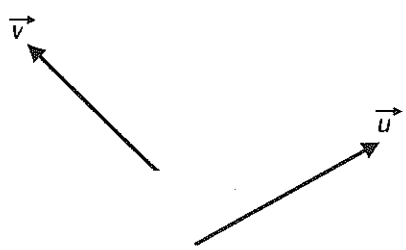


11. You can also subtract vectors $\vec{u}-\vec{v}$ by reversing the direction of $\vec{v}$ and then performing an addition problem. This is similar to subtracting scalars on a number line, because subtraction on a number line is the same as addition of a negative of a number. For example,

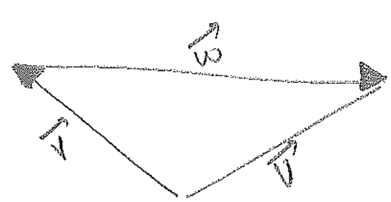
$$5-3 = 5+(-3)$$

$$6-(-4) = 6+(-(-4)) = 6+4$$

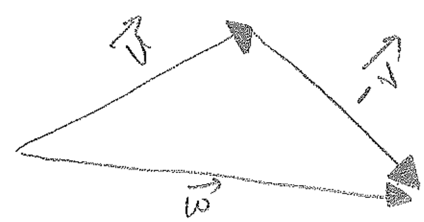
Try both methods on the vectors below, and write down which method you prefer.



I



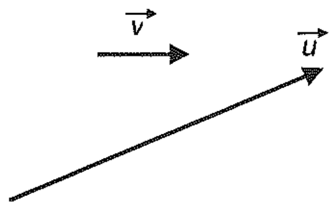
II



$\vec{u}+(-\vec{v})=\vec{w}$

$\vec{u}-\vec{v}=\vec{w}$

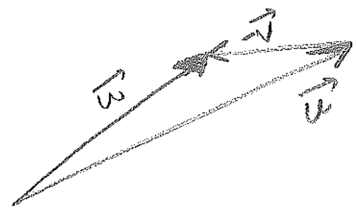
12. You are walking on a plane. On the picture below, vector $\overrightarrow{u}$ shows your velocity with respect to an outside viewer and vector $\overrightarrow{v}$ shows your velocity with respect to the plane. Show the velocity of the plane with respect to an outside viewer.



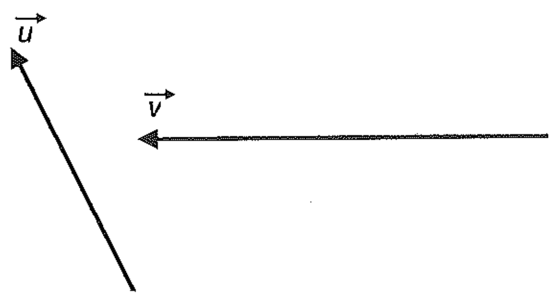
$\vec{u} = \vec{v} + \vec{w}$

$\vec{w} = \vec{u} - \vec{v}$

$\vec{w}$ is the velocity of the plane



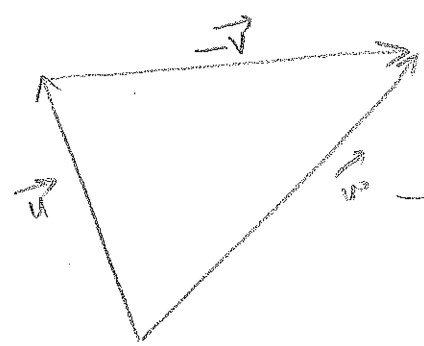
13. You are walking on a boat, where vector $\overrightarrow{u}$ below shows your velocity with respect to an outside viewer and vector $\overrightarrow{v}$ shows the velocity of the boat with respect to an outside viewer. Show the your velocity with respect to the boat.



$\vec{u} = \vec{v} + \vec{w}$

$\vec{w}$ is the velocity of the person on the boat

So, $\vec{w} = \vec{u} - \vec{v}$

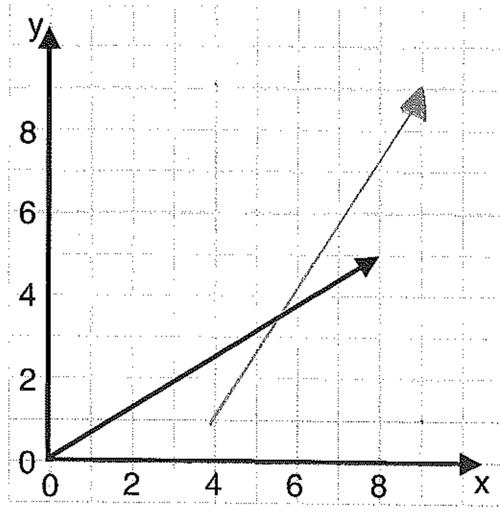


$\longrightarrow$ $\vec{w} = \vec{u} - \vec{v}$

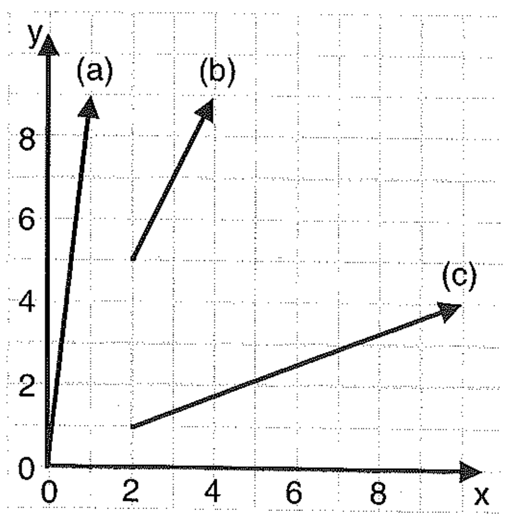
Vectors on a coordinate plane

It is common to draw vectors on a coordinate plane. Similar to how a point is denoted by (x, y), a vector can be specified by its coordinates $< x, y >$. A vector $< 8, 5 >$ moves in the x-direction by 8 units and the y-direction by 5 units. Note that it is customary to draw vectors starting at the origin, but this does not always have to be the case.

14. Below is the vector $< 8, 5 >$. Draw vector $< 5, 8 >$ by starting at point $(4, 1)$.

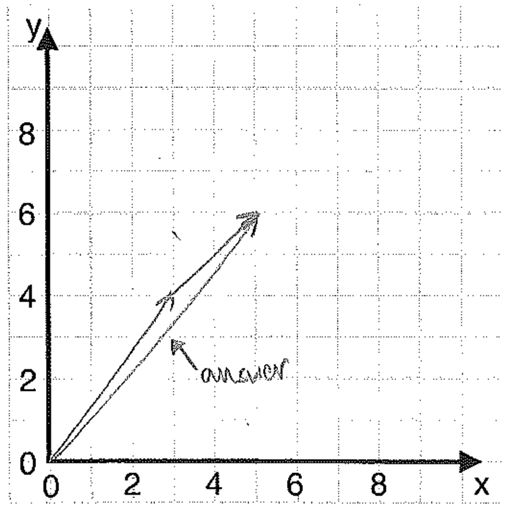


15. Below are three vectors on the coordinate plane. Write down the components of each vector using $< x, y >$ notation.



a: < 1, 9 >
b: < 2, 4 >
c: < 8, 3 >

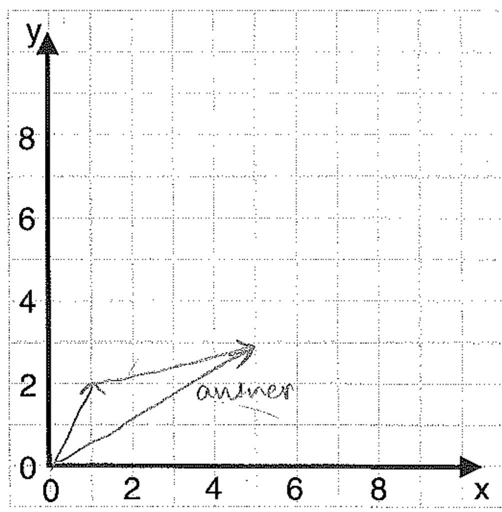
16. Using the coordinate plane below, draw the vector that results from adding vector $< 3, 4 >$ with vector $< 2, 2 >$. (*Hint:* first draw vector $< 3, 4 >$ starting at the origin).



What is the resulting vector, using the notation $< x, y >$?

$<5, 6>$

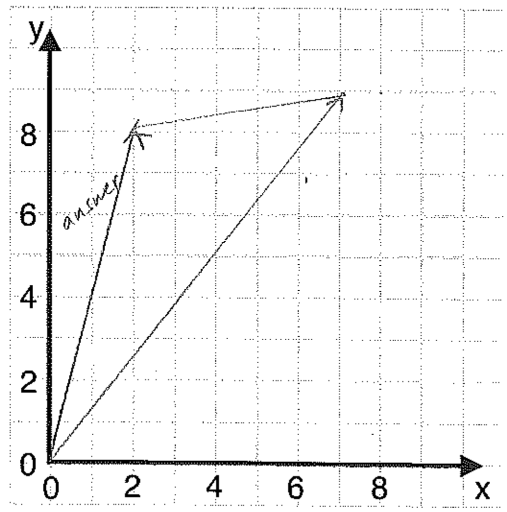
17. Use the coordinate plane below to draw the vector resulting from $< 1, 2 > + < 4, 1 >$.



What is the resulting vector, using the form $< x, y >$?

$<5, 3>$

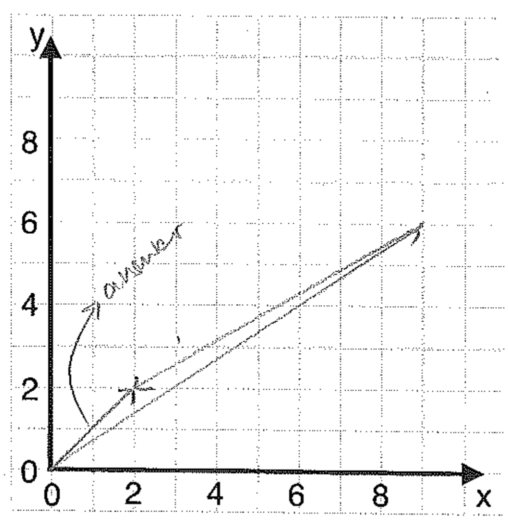
18. Use the coordinate plane below to draw the vector resulting from $< 7, 9 > - < 5, 1 >$.



What is the resulting vector, using the form $< x, y >$?

< 2, 8 >

19. Use the coordinate plane below to draw the vector resulting from $< 9, 6 > - < 7, 4 >$.



What is the resulting vector, using the form $< x, y >$?

< 2, 2 >

20. Using your answers from the previous problems, determine the vectors that result from the following operations without drawing the vectors.

(a) $< 15, 8 > + < 5, 21 > =$ $<20, 29>$

(b) $< 9, 10 > - < 10, 9 > =$ $<-1, 1>$

(c) $< a, b > + < c, d > =$ $<a+c, b+d>$

(d) $< a, b > - < c, d > =$ $<a-c, b-d>$