NUMBER BASES: PART II

BEGINNERS 05/10/2015

When the bases are powers of each other, we can convert numbers from one base to another by breaking them apart.

Example 1. What is 312_4 in base 2?

(1) Expand 312_4 by writing it in terms of powers of 4:

$312_4 = \underline{3} \times 4^2 + \underline{1} \times 4^1 + \underline{2} \times 4^0$

(2) Write each term from the expression above in base 2:

(a) $\underline{3} \times 4^2$

$= \underline{48}$ (in base 10)

$= \underline{1} \times 2^5 + \underline{1} \times 2^4 + \underline{0} \times 2^3 + \underline{0} \times 2^2 + \underline{0} \times 2^1 + \underline{0} \times 2^0$

(b) $\underline{1} \times 4^1$

$= \underline{4}$ (in base 10)

$= \underline{0} \times 2^3 + \underline{1} \times 2^2 + \underline{0} \times 2^1 + \underline{0} \times 2^0$

(c) $\underline{2} \times 4^0$

$= \underline{2}$ (in base 10)

$= \underline{1} \times 2^1 + \underline{0} \times 2^0$

(3) Add all of the terms above to get your final answer:

$312_4 = \underline{3} \times 4^2 + \underline{1} \times 4^1 + \underline{2} \times 4^0$

$= \underline{1} \times 2^5 + \underline{1} \times 2^4 + \underline{0} \times 2^3 + \underline{1} \times 2^2 + \underline{\quad} \times 2^1 + \underline{\quad} \times 2^0$

$= \underline{110110}$ (in base 2)

Example 2. What is 73_8 in base 2?

(1) Expand 73_8 by writing it in terms of powers of 2:

$73_8 = \underline{7} \times 8^1 + \underline{3} \times 8^0$

(2) Write each term from the expression above in base 2:

(a) $\underline{7} \times 8^1$

$= \underline{56}$ (in base 10)

$= \underline{1} \times 2^5 + \underline{1} \times 2^4 + \underline{1} \times 2^3 + \underline{0} \times 2^2 + \underline{0} \times 2^1 + \underline{0} \times 2^0$

(b) $\underline{3} \times 8^0$

$= \underline{3}$ (in base 10)

$= \underline{0} \times 2^2 + \underline{1} \times 2^1 + \underline{1} \times 2^0$

(3) Add all of the terms above to get your final answer:

$73_8 = \underline{7} \times 8^1 + \underline{3} \times 8^0$

$= \underline{1} \times 2^5 + \underline{1} \times 2^4 + \underline{1} \times 2^3 + \underline{0} \times 2^2 + \underline{1} \times 2^1 + \underline{1} \times 2^0$

$= \underline{111011}$ (in base 2)

(1) Using the method above, convert 21_4 to base 2.

$21_4 = 2 \times 4^1 + 1 \times 4^0$

$= 8 + 1$

$= 2^3 + 2^0$

$21_4 = 1001$

(2) Using the method above, convert 271_8 to base 2.

$271_8 = 2 \times 8^2 + 7 \times 8^1 + 1 \times 8^0$

$= 128 + 56 + 1$

$= 2^6 + (2^5 + 2^4 + 2^3) + 2^0$

$= 10111001$

(3) Using the method above, convert 68_9 to base 3.

$68_9 = 6 \times 9^1 + 8 \times 9^0$

$= 54 + 8$

$= 2 \times 3^3 + 2 \times 3^1 + 2 \times 3^0$

$= 2022$

(4) Suppose you are given a number written in base 4. Describe in your own words how you would get the digits in base 2 from the digits written in base 4. Do you notice any patterns that would make the process faster?

Convert the base-4 number to base 10.
Convert the base-10 number to base 2.

Directly convert the digits in base 4 to base 2.

(5) Can you use this pattern to quickly convert BAD_{16} to base 2?

$BAD_{16} = [B][A][D]_{16}$
$= [1011][1010][1101]_2$
$= 101110101101$

Convert each digit in base 16 to base 2.

(6) How about $CAFE_{16}$ to base 4? (Hint: Remember that $16 = 4^2$)

$CAFE_{16} = [C][A][F][E]_{16}$
$= [30][22][33][32]_4$
$= 30223332$

(7) Can you use this pattern to do the opposite and convert 33000031_4 to base 16? (Hint: Remember that $16 = 4^2$)

$[33][00][00][31]_4 = [F][0][0][D]$

(8) In question (4), you described how to convert numbers from a higher base to a lower base that are powers of each other. Can we reverse the process you described in the question (4) above to convert from a lower base to a higher base? Suppose you are given a number written in base 2. Describe in your own words how you would get the digits in base 4 from the digits written in base 2.

Directly convert pairs of digits in base 2 to equivalent digits in base 4.

(9) Can you use this method to convert a number written in base 125 to base 5? Why or why not? (Hint: Remember that $125 = 5^3$)

Yes.

(10) Can you use this method to convert a number written in base 5 to base 2? Why or why not?

5 and 2 are not powers of each other.