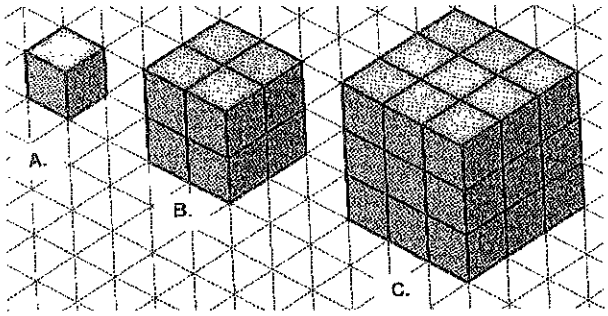
WARM-UP: SURFACE AREAS & VOLUMES

BEGINNERS 05/03/2015

(1) Each individual cube below has a side length of 1 cm.



(a) What are the volumes of solids A, B, and C?

A: $1\,cm^3$ B: $2^3 = 8\,cm^3$ C: $3^3 = 27\,cm^3$

(b) How much bigger is the volume of solid B than A?

8 times

(c) How much bigger is the volume of solid C than A?

27 times

(d) If solid D is made with a side length of 8 cm, how much bigger is the volume of solid D compared to B?

$8^3/2^3 = (8/2)^3 = 4^3 = 64$ times

(e) What are the surface areas of solids A, B, and C?

A: $6(1^2) = 6\,cm^2$; B: $6(2^2) = 24\,cm^2$ C: $6(3^2) = 54\,cm^2$

(f) How much bigger is the surface area of solid B than A?

4 times

(g) How much bigger is the surface area of solid C than A?

9 times

(h) If solid D is made with a side length of 18 cm, how much bigger is the surface area of solid D when compared to C?

$\frac{6(18^2)}{6(3^2)} = (\frac{18}{3})^2 = 6^2 = 36$ times

NUMBER BASES

BEGINNERS 05/03/2015

The numeral system we use in everyday life is base ten. This means that we use ten digits:

$$0, 1, 2, 3, 4, 5, 6, 7, 8, 9$$

and that the value of a digit depends on its position in a number. For example,

$$2015 = 2 \cdot 10^3 + 0 \cdot 10^2 + 1 \cdot 10^1 + 5 \cdot 10^0$$

However, numbers can be written in any number base $n \geq 2$.

- Base n uses digits $0, 1, 2, ..., n-1$.
- If $n > 10$, letters are used instead of digits starting with the digit representing 10.

(1) Fill the table below.

Number/Base	2	3	4	7	8	16
0	0	0	0	0	0	0
1	1	1	1	1	1	1
2	10	2	2	2	2	2
3	11	10	3	3	3	3
4	100	11	10	4	4	4
5	101	12	11	5	5	5
6	110	20	12	6	6	6
7	111	21	13	10	7	7
8	1000	22	20	11	10	8
9	1001	100	21	12	11	9
10	1010	101	22	13	12	A
11	1011	102	23	14	13	B
12	1100	110	30	15	14	C
13	1101	111	31	16	15	D
14	1110	112	32	20	16	E
15	1111	120	33	21	17	F
16	10000	121	100	22	20	10

(2) Consider the table on the previous page.

(a) Write n in base n for any n:

10

(b) Write n^2 in base n for any n:

100

(c) Write n^{2015} in base n for any n:

1 0000000 . . . . 00 (2015 zeroes)

(3) Fill out the table below.

Number/Base	2	3	4	8	9	16
22	10110	211	112	26	24	16
38	100110	1102	212	46	42	26
54	110110	2000	312	66	60	36
100	1100100	10201	1210	144	121	64
232	11101000	22121	3220	350	277	E8
1000	1111101000	1101001	33220	1750	1331	3E8
2015	11111011111	2202122	133133	3737	2678	7DF

There are two ways to go to any base from base 10:

(A) The Greedy Algorithm: determines notation from left to right.

- Find the highest power of the base that fits into the given number (can fit more than once!)
- Subtract this power from the number. You can subtract this power as many times as it fits.
- Repeat with the new number.

Example: What is 197_{10} in base 9?
$\mathbf{2} \cdot 9^2 = 162$, $197 - 162 = 35$, $35 < 9^2$, 2 is the left-most digit
$\mathbf{3} \cdot 9^1 = 27$, $35 - 27 = 8$, $8 < 9^1$, 3 is the next digit
$\mathbf{8} \cdot 9^0 = 8$, 8 is the right-most digit
$197_{10} = 238_9$

(B) The Division Algorithm: determines notation from right to left.

- Divide the number by the base and find the remainder.
- Divide the quotient by the base and find the remainder.
- Repeat until you can no longer divide the quotient by the base.

Example: What is 197_{10} in base 9?
$197 : 9 = 21$ Remainder **8**, 8 is the right-most digit
$21 : 9 = 2$ Remainder **3**, 3 is the next digit
$2 : 9 = 0$ Remainder **2**, 2 is the left-most digit
$197_{10} = 238_9$

(1) Solve the following problems using the greedy algorithm:

(a) What is 27_{10} in base 2?

$16 = 2^4$ $\quad 27-16=11$ $\quad 1$

$8 = 2^3$ $\quad 11-8=3$ $\quad 1$

$4 = 2^2$ $\quad 3-4<0$ $\quad 0$

$2 = 2^1$ $\quad 3-2=1$ $\quad 1$ $\quad$ So, 11011.

$1 = 2^0$ $\quad 1-1=0$ $\quad 1$

(b) What is 121_{10} in base 3?

$81 = 3^4$ $\quad 121-81=40$ $\quad 1$

$27 = 3^3$ $\quad 40-27=13$ $\quad 1$

$9 = 3^2$ $\quad 13-9=4$ $\quad 1$

$3 = 3^1$ $\quad 4-3=1$ $\quad 1$ $\quad$ So, 11111.

$1 = 3^0$ $\quad 1-1=0$ $\quad 1$

(c) What is 284_{10} in base 8?

$4 \times 8^2 = 4 \times 64 = 256$ $\quad 284-256=28$ $\quad 4$

$3 \times 8^1 = 2 \times 8 = 16$ $\quad 28-24=4$ $\quad 3$

$4 \times 8^0 = 4 \times 1 = 4$ $\quad 4-4=0$ $\quad 4$

So, 434

(d) What is 299_{10} in base 12?

$2 \times 12^2 = 2 \times 144 = 288$ $\quad 299-288=11$

$12^0 \times 11 = 11 \times 1 = 11$ $\quad 11-11=0$

2
11 ↳ B in base 12.

Because we skipped 12^1, we will put a 0.

So, 20B.

(e) What is 342_{10} in base 7?

$6 \times 7^2 = 6 \times 49 = 294$ $\quad 342-294=48$ $\quad 6$

$6 \times 7^1 = 6 \times 7 = 42$ $\quad 48-42=6$ $\quad 6$

$6 \times 7^0 = 6 \times 1 = 6$ $\quad 6-6=0$ $\quad 6$

So, 666

(2) Solve the following problems using the division algorithm:

(a) What is 57_{10} in base 4?

57 : 4 = 14 R 1
14 : 4 = 3 R 2
3 : 4 = 1 R 3.

So, 321.

(b) What is 275_{10} in base 7?

275 : 7 = 39 R 2
39 : 7 = 5 R 4
5 : 7 = 0 R 5

So, 542.

(c) What is 143_{10} in base 8?

143 : 8 = 17 R 7
17 : 8 = 2 R 1
2 : 8 = 0 R 2

So, 217

(d) What is 342_{10} in base 9?

342 : 9 = 38 R 0
38 : 9 = 4 R 2
4 : 9 = 0 R 4

So, 420.

(e) What is 432_{10} in base 15?

432 : 15 = 28 R 12
28 : 15 = 1 R 13
1 : 15 = 0 R 1

12 is C
13 is D
1DC.

(3) Which method do you prefer? Explain.

When converting numbers with base greater than 10, we must remember that letters are used instead of digits.

(A) Consider the conversion between base 16 and base 10.

Example: What is $1FDA_{16}$ in base 10?
$1FDA_{16} = \mathbf{1} \cdot 16^3 + \mathbf{15} \cdot 16^2 + \mathbf{13} \cdot 16^1 + \mathbf{10} \cdot 16^0$
$1FDA_{16} = 1 \cdot 4096 + 15 \cdot 256 + 13 \cdot 16 + 10$
$1FDA_{16} = 8154_{10}$

(1) Computer the following powers of 16.

(a) $16^1 =$ 16

(b) $16^2 =$ 256

(c) $16^3 =$ 4096

(d) $16^4 =$ 65536

(2) Convert the following numbers between base 16 and base 10:

(a) What is 1984_{10} in base 16?

1984 : 16 = 124 R 0
124 : 16 = 7 R 12 12 is C in base 16
7 : 16 = 0 R 7

7C0

(b) What is 4813_{10} in base 16?

4813 : 16 = 300 R 13 D
300 : 16 = 18 R 12 C
18 : 16 = 1 R 2 2
1 : 16 = 0 R 1 1

So, 12CD

(c) What is 1984_{16} in base 10?

$1 \times 16^3 + 9 \times 16^2 + 8 \times 16 + 4 \times 16^0$
$= 4096 + 9 \times 256 + 8 \times 16 + 4$
$= 4096 + 2304 + 128 + 4 = 6532$

(d) What is $17EC_{16}$ in base 10?

$1 \times 16^3 + 7 \times 16^2 + 14 \times 16 + 12 \times 16^0$
$= 4096 + 7 \times 256 + 14 \times 16 + 12$
$= 4096 + 1792 + 224 + 12 = 6124$