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Boolean algebra and logic gates

Recall that Boolean algebra, a.k.a. the algebra of logic, is the algebra of statements that can take either the value 1 (True) or 0 (False). Simple statements can be combined into composite ones by using the operations of logical addition (**Or**), logical multiplication (**And**), and negation (**Not**).

Problem 1 Find the value of the composite statement $A + B \neg C$ for the simple statements A , B , and C below.

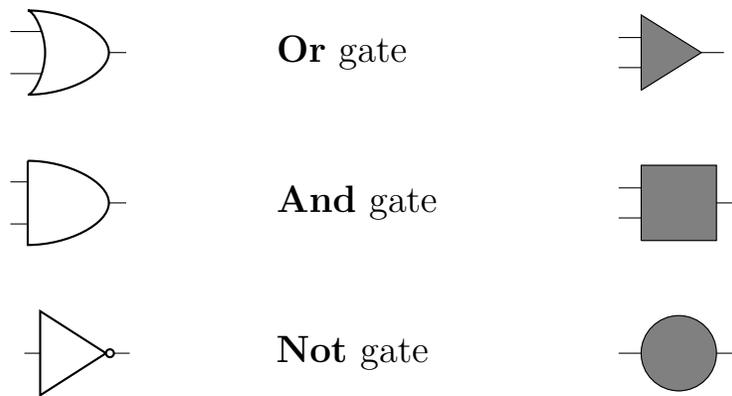
A: California is a part of kangaroo's natural habitat.

B: Math Kangaroo is at home at UCLA.

C: Statements A and B contradict each other.

$$A + B \neg C =$$

Logic gates are hardware realizations of the Boolean algebra operations that power our computers and smartphones. On the left-hand side of the table below you will see the logic gates' symbols standardly used in the US. On the right-hand side, you will see their easy to draw equivalents that we will use in the problems below.



The **Or** gate operates as follows. If at least one of the inputs has current, denoted as 1 on the picture below, then the output has current as well. If none on the inputs has current, absence of the current denoted by 0, then neither has the output.



As you can see, the **Or** gate is simply a hardware implementation of the truth table for the logical addition (please see the

table on the next page).

| A | B | $A + B$ |
|-----|-----|---------|
| 0 | 0 | 0 |
| 1 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 1 | 1 |

Problem 2 Fill out the truth table for the logical multiplication **And**.

| A | B | $A \times B$ |
|-----|-----|--------------|
| 0 | 0 | |
| 1 | 0 | |
| 0 | 1 | |
| 1 | 1 | |

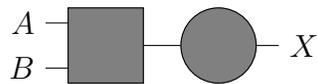
Problem 3 Mark the inputs and outputs of the **And** logic gate corresponding to the truth table from Problem 2.



The **Not** gate, also known as an *inverter*, always inverts the incoming signal.

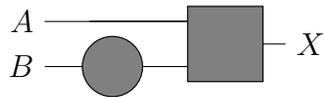


Problem 4 What is the value of the output X for the input values $A = 1$ and $B = 0$?



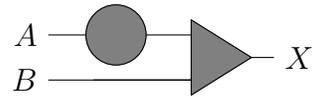
$X =$

Problem 5 Find the value of the output X for the following input values.



- $A = 0, B = 1, X =$
- $A = 1, B = 1, X =$

Problem 6 Find the value of the output X for the input values $A = 0$ and $B = 0$.



$X =$

Let us recall the De Morgan's laws.

$$\neg(A + B) = \neg A \times \neg B \qquad \neg(A \times B) = \neg A + \neg B$$

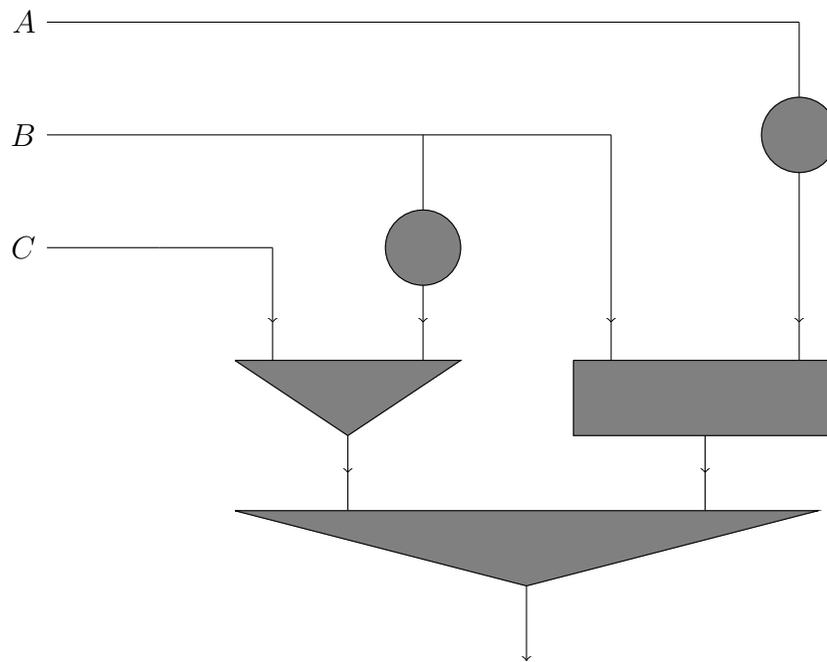
Problem 7 Use logic gates circuits to prove the first of the De Morgan's laws.

Problem 8 Use logic gates circuits to prove the second of the De Morgan's laws, $\neg(A \times B) = \neg A + \neg B$.

Problem 9 Realize the **And** gate by means of a circuit that has inverters and **Or** gates only.

Problem 10 Realize the **Or** gate by means of a circuit that has inverters and **And** gates only.

Problem 11 What Boolean algebra formula is implemented by the following circuit?

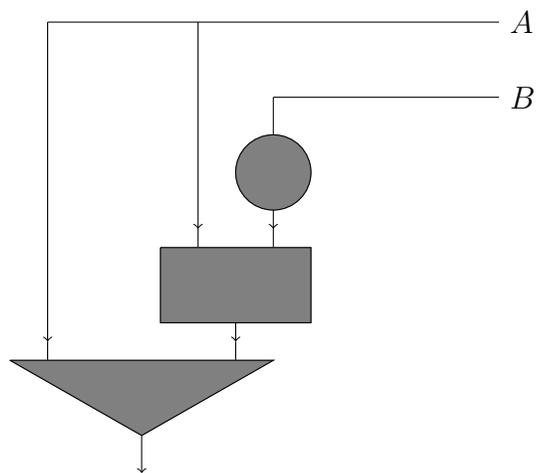


Problem 12 Draw the logic gates circuit that implements the following Boolean algebra formula.

$$\neg AB\neg(A + \neg B)$$

Hint: it helps to simplify the formula first.

Problem 13 Simplify the following circuit.



Problem 14 *In a two-story townhouse, there is one electric light over the stairs from the first to the second floor. The light has two switches, switch A at the bottom and switch B at the top of the stairs. Design the circuit satisfying the following requirements.*

- *When someone at the bottom of the stairs turns A on, the light is on. B is assumed to be in the off position.*
- *With A on, switching B to the on position turns off the light.*
- *If a new person enters the house and flips A to the off position, with B still in the on mode, the light turns on.*
- *With A in the off position, the person upstairs can turn off the light by switching B to the off mode.*

Problem 15 *Design a circuit for adding two one-digit binary numbers.*

Problem 16 *There are three CNC¹ machines, A, B, and C, and two power generators, X and Y, in a metal shop. The generator X has enough power to run any single one of the machines, but not enough to simultaneously run two. Y has enough power to run any two of the machines, but not all three of them at the same time. Design a circuit that does the following.*

- *When one machine is working, X is on, Y is off.*
- *When two machines are working, X is off, Y is on.*
- *When all the three machines are working, both X and Y are on.*

¹Computerized Numerical Control

Problem 17 *Design a circuit for adding three one-digit binary numbers.*

Problem 18 *Think of a task that you would like to automatize.
Design the corresponding logic gates circuit.*