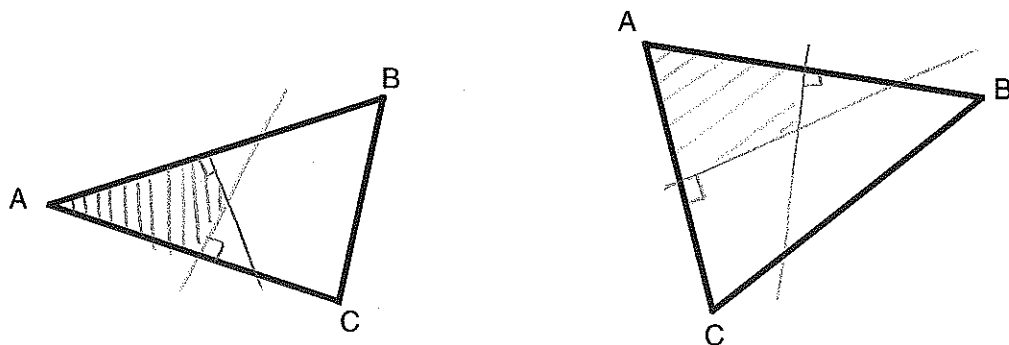# Geometry Part II: 3D Projections and Solids
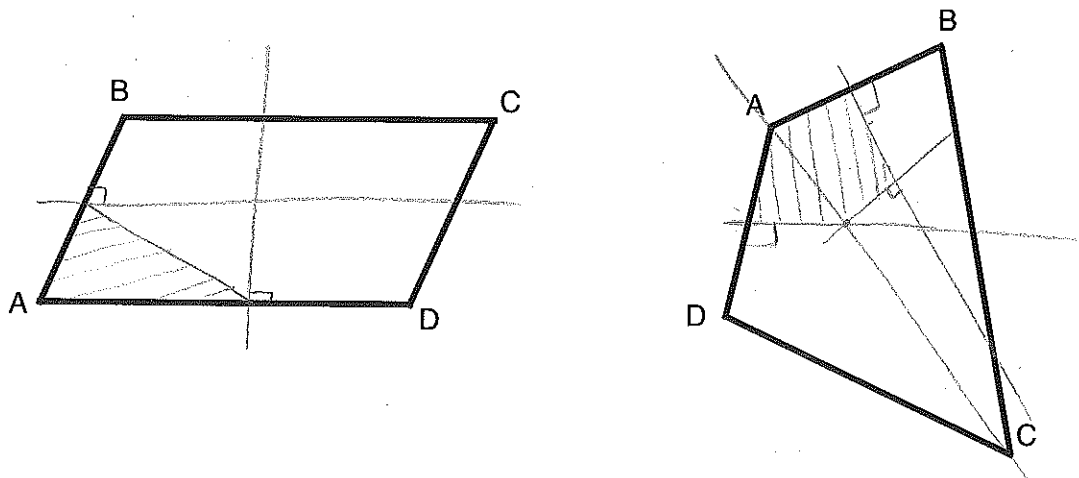
April 26, 2015

Warm up Problem

Shade in all points in triangles ABC that are closer to vertex A than any other vertex.

Shade in all points in quadrilaterals $ABCD$ that are closer to vertex A than any other vertex. Use perpendicular bisectors.
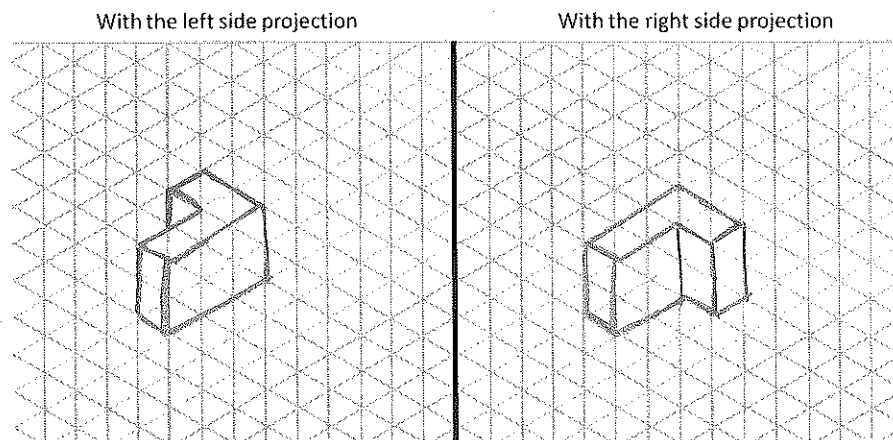
3D Projections

Take a look at the solid structure built by putting two identical cubes next to each other. The diagram on the left shows 2D projections (top, front and the right side projection). The diagram on the right side is an isometric view. The isometric projection is a method of drawing a 3D shape on the plane in such a way that the angles between the three main directions (front $\leftrightarrow$ back, left $\leftrightarrow$ right, up $\leftrightarrow$ down) are all equal to 120°.
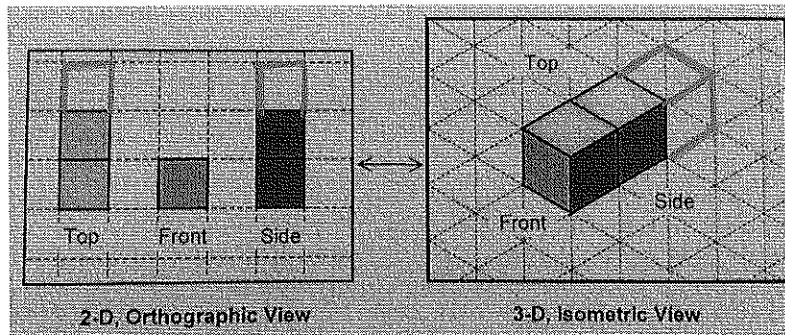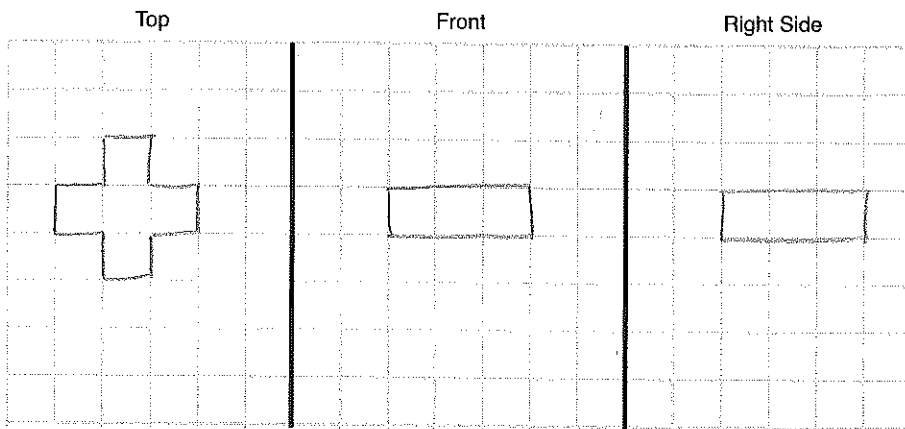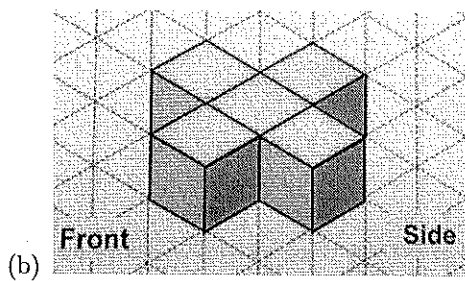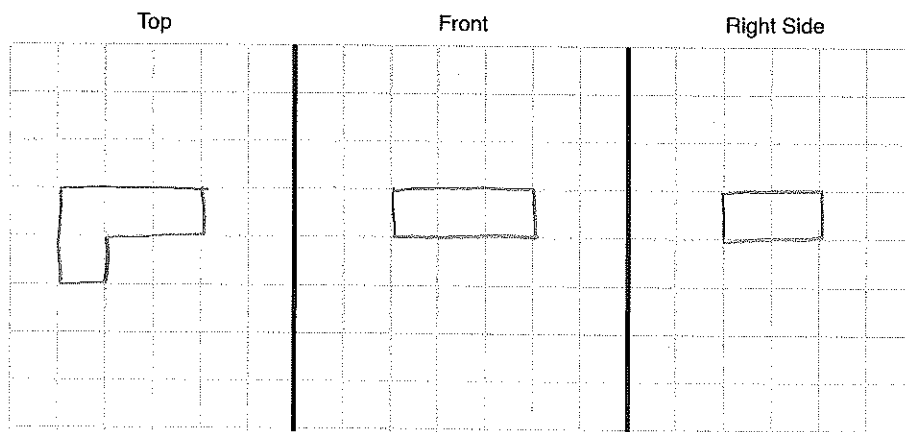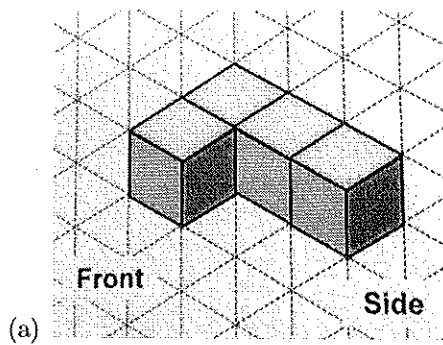

The 3D image shows an isometric view. Take your time looking at the isometric view, and try to understand how it works.
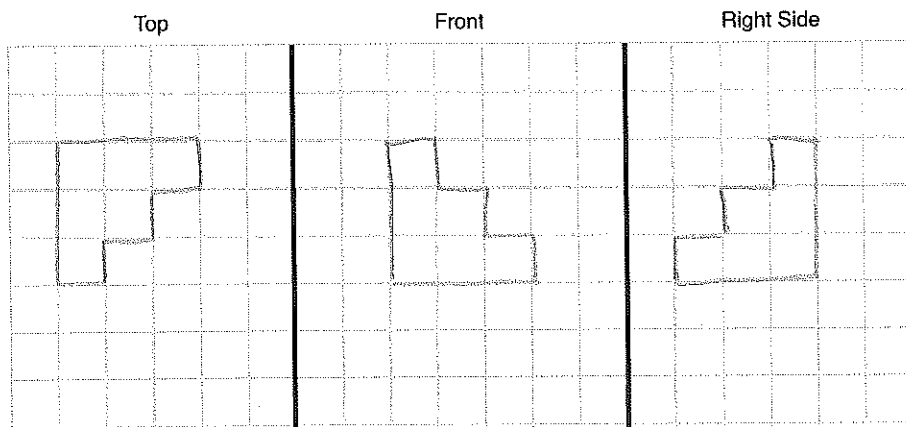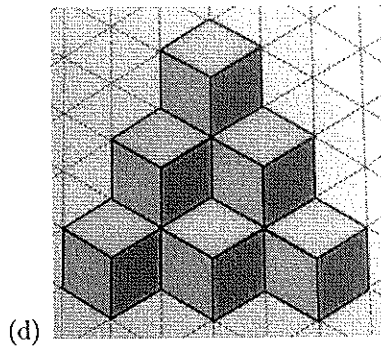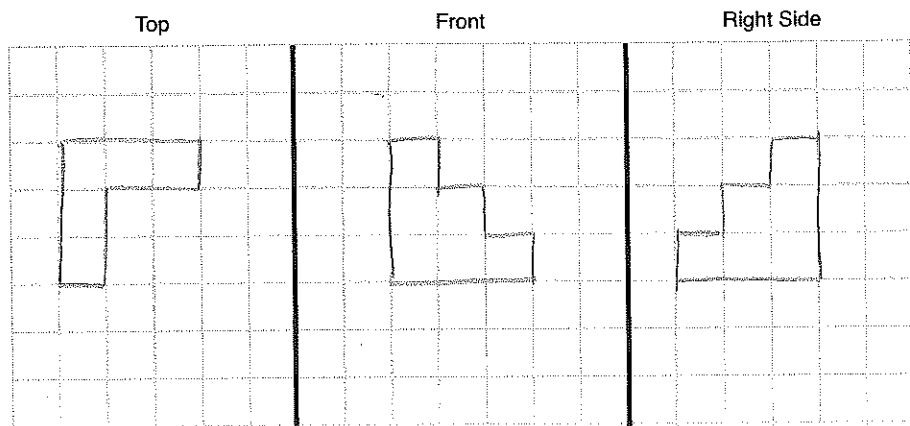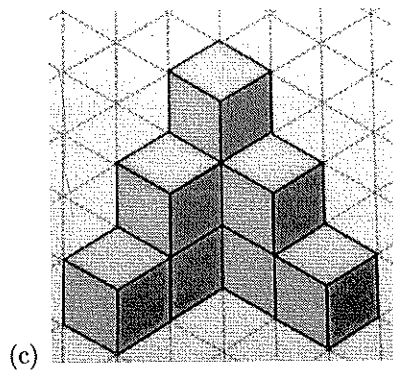
1. Draw a third cube attached to the back of the 3D figure above. Please change the projections accordingly.

2. We will use the right side projections. Note that right and left side projections are usually not the same. Suppose you are given the top, front and right side projections, and you build a solid using these projections. However, you mistakenly use the right side projection as the left side projection. What would be the difference between the solid you built and the solid you were supposed to build? Draw an example of this phenomenon.
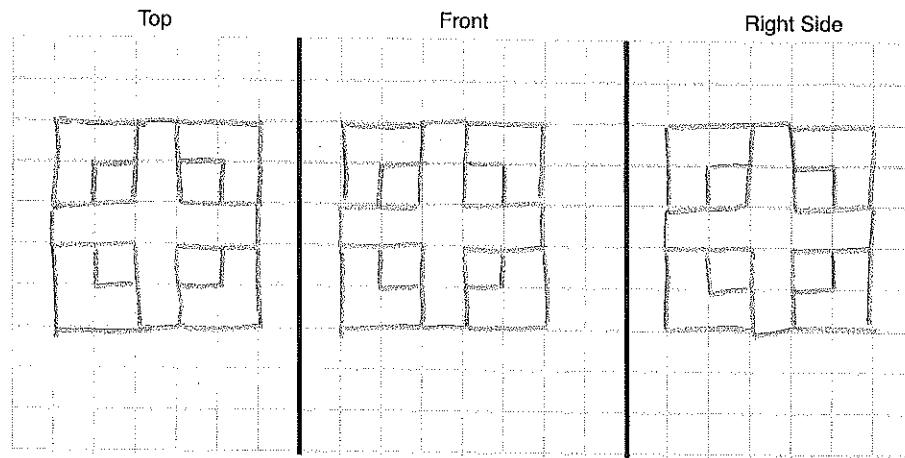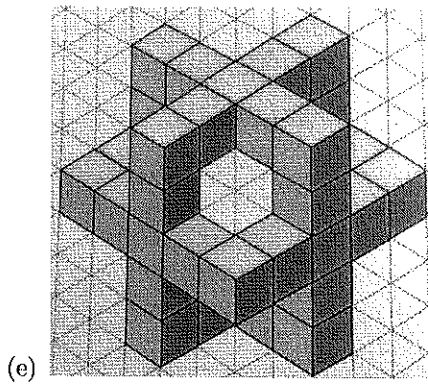
MIRROR
IMAGES

With the left side projection   With the right side projection

2

3. Draw the front, side and top views of the given solids. Remember that the 3D solids are drawn on isometric paper, while the 2-D projections are drawn on orthographic paper.
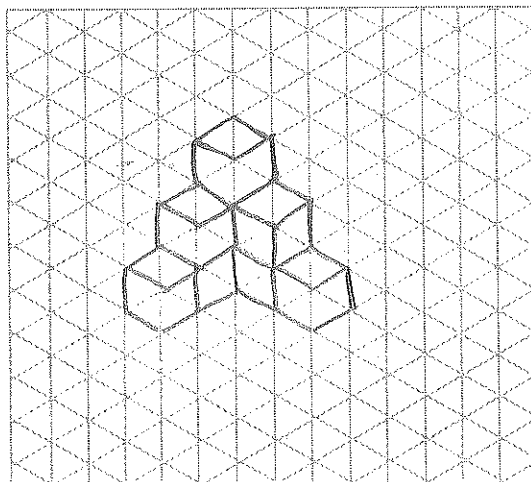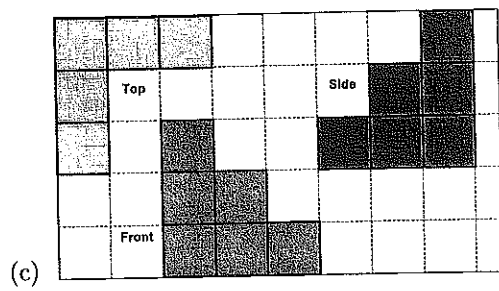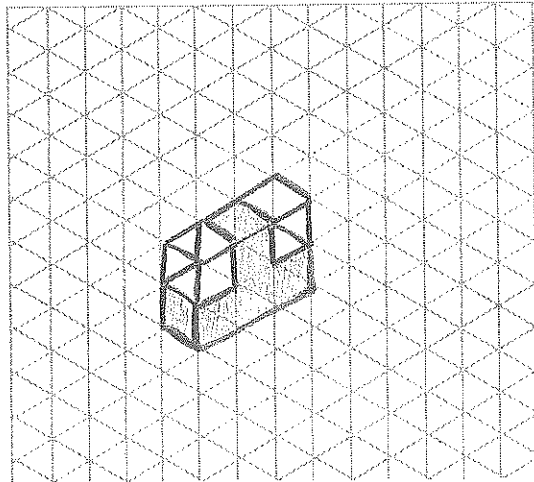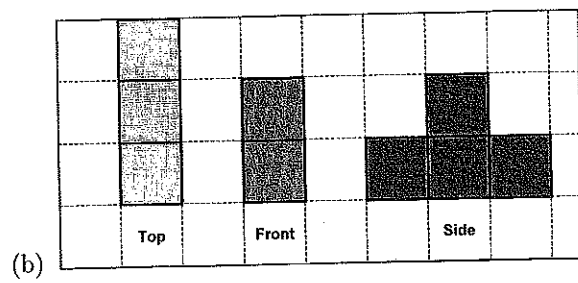
(a)

Top	Front	Right Side

(b)

Top	Front	Right Side

(c)

Top	Front	Right Side

(d)

Top	Front	Right Side

4

(e)

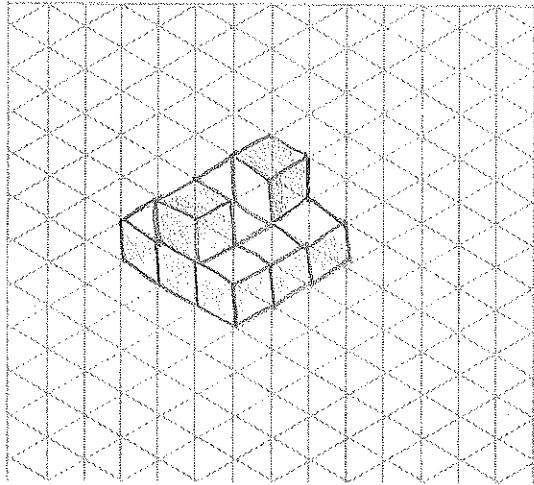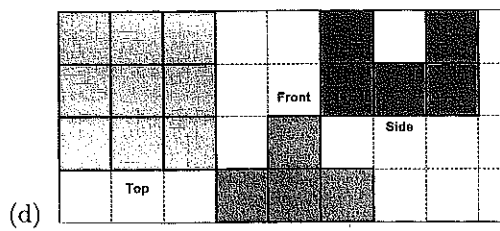| Top	Front	Right Side

4. Given the following 2D projections, draw the solid structures on isometric paper.

Top   Front   Side

(a)

(b)

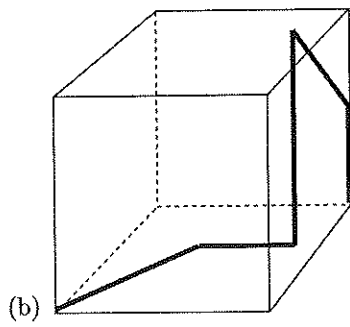Top   Front   Side

(c)

Top   Side

Front

(d)
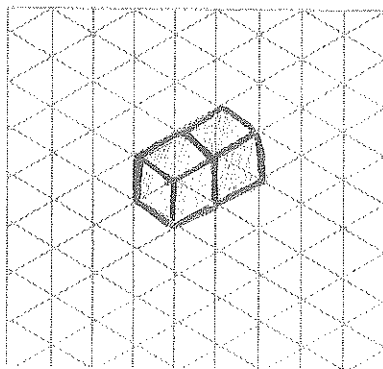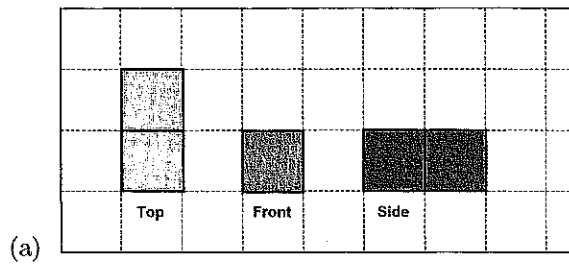
5. The following cubes represent aquariums. Imagine that the darkened line shows the path that a fish takes. Draw what you think the top, front and right-side projections for each path is.
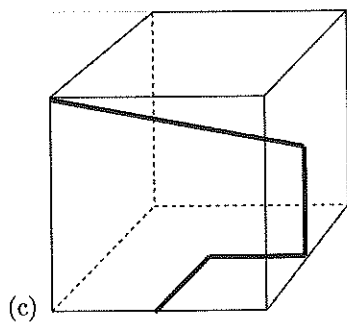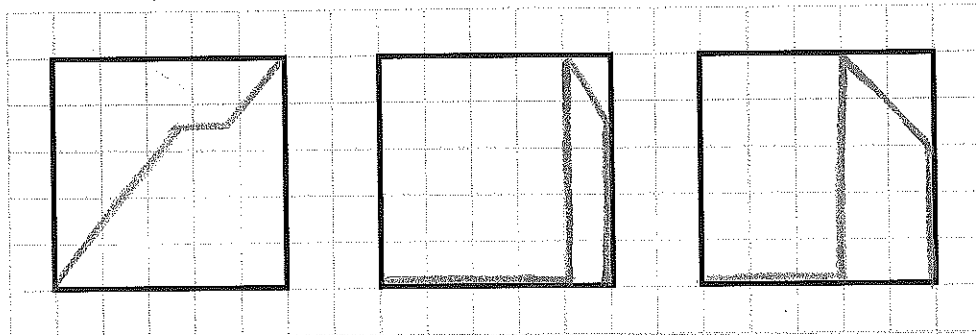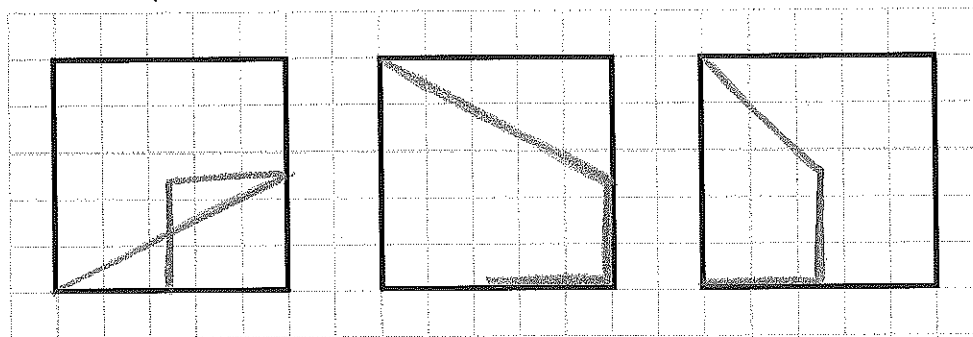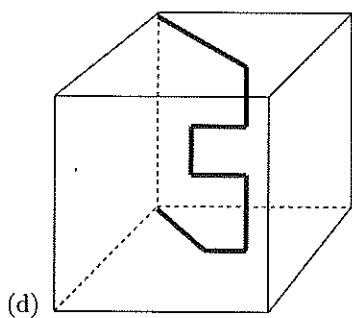
(a)

Top	Front	Right Side

(b)

Top	Front	Right Side

(c)

Top	Front	Right Side

(d)
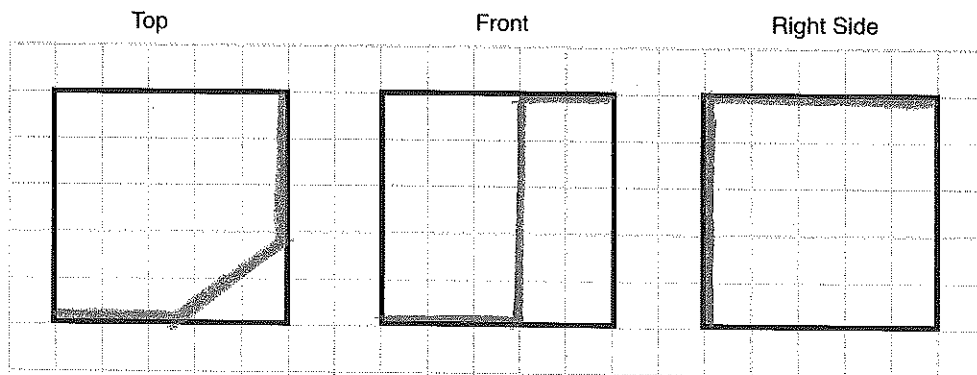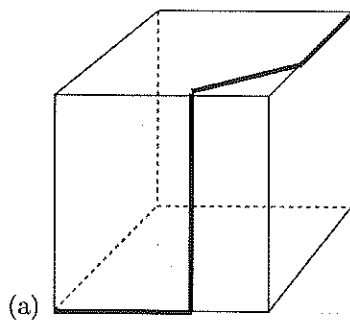
Top	Front	Right Side
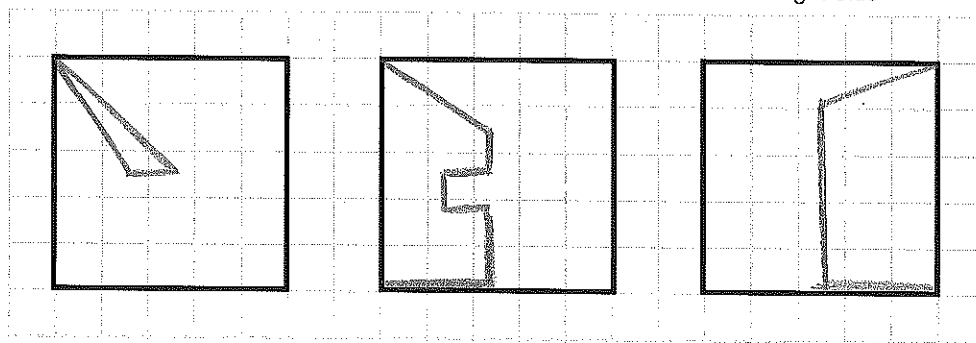
6. If three people look at the aquarium from the top, the front and the right-side, respectively, they see that the fish take takes the following paths. Draw the path that the fish actually takes in the tank on the isometric paper given below. (Hint: First, draw a $3 \times 3 \times 3$ cube.)
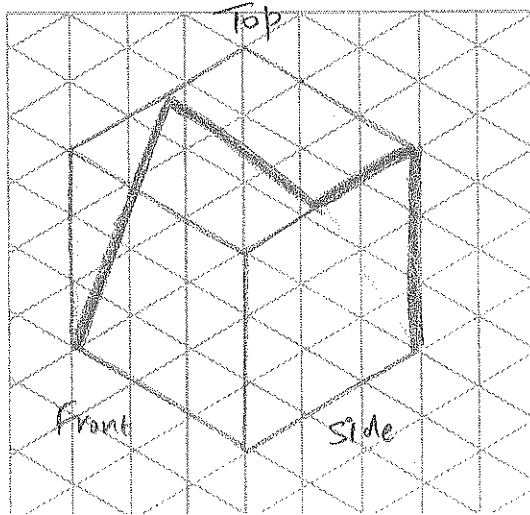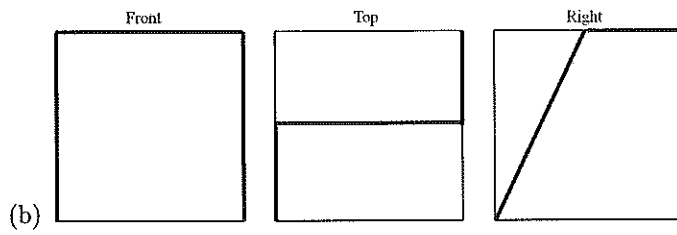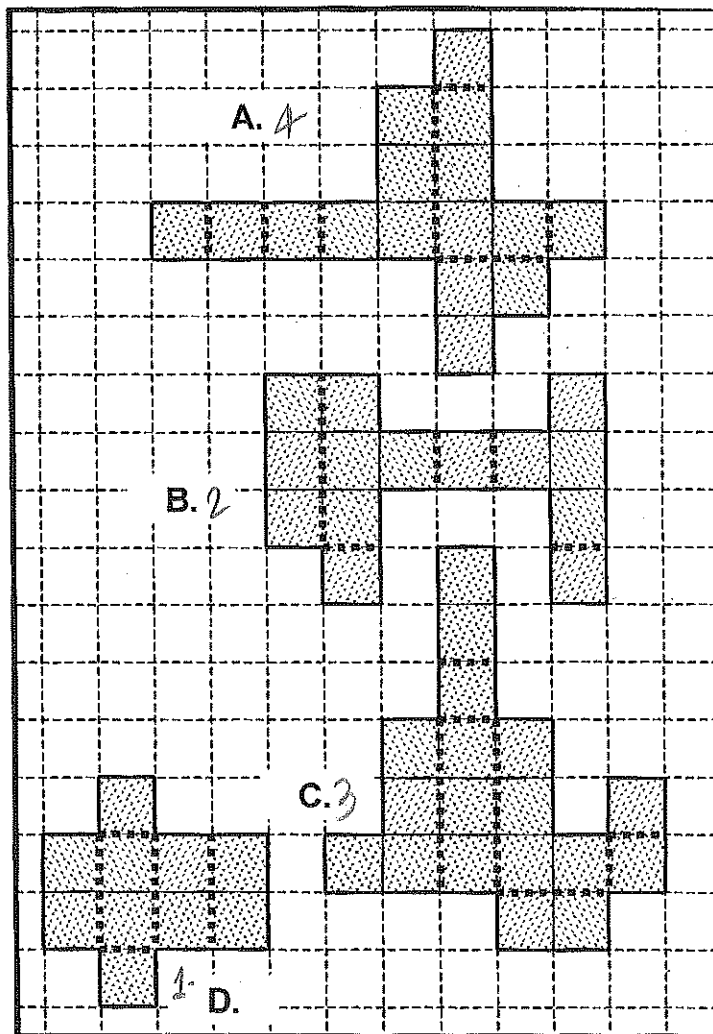
(a)

Front   Top   Right   Left

(b)

Front | Top | Right

Top

Front   Side

(c)

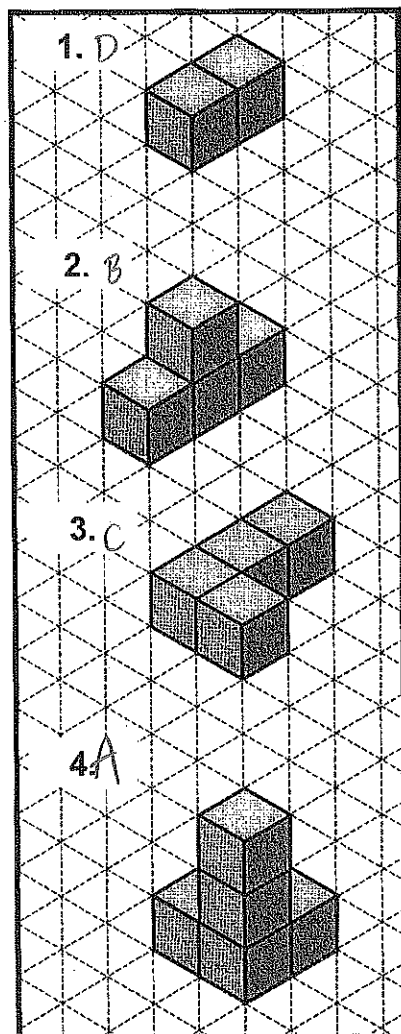Front | Top | ~~Right~~ Left


Top

Front   Side
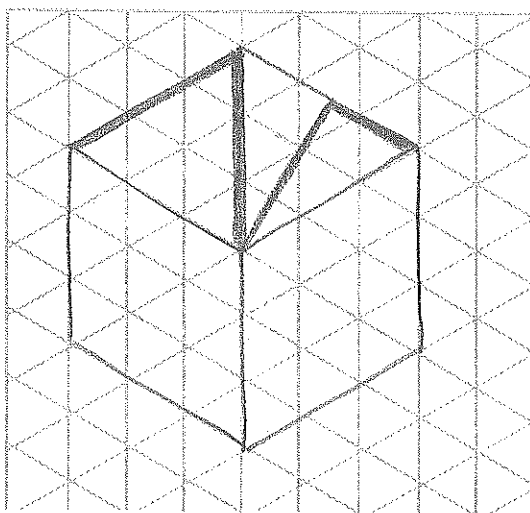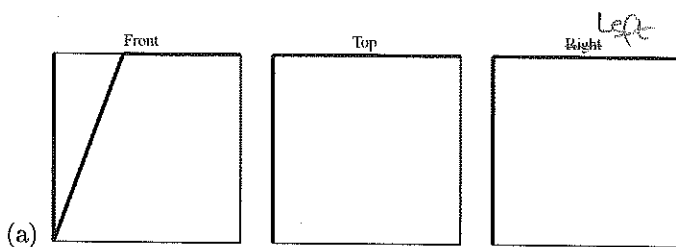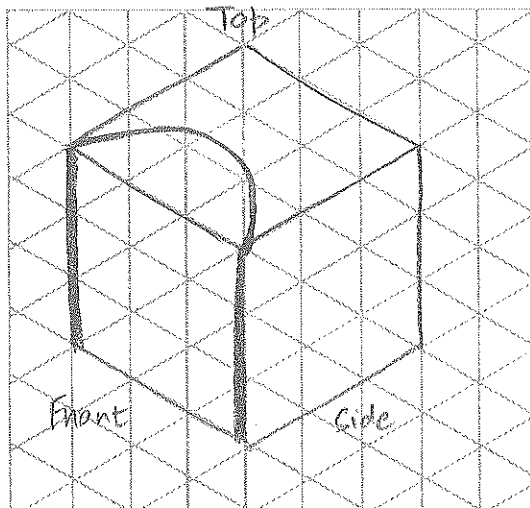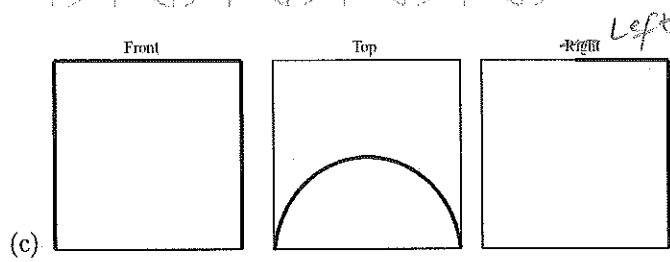
Solid structures

1. Look at the drawing of the cube shown below. If you were to cover the surface with paper, you would create a "jacket." For example, figure Y is the jacket for figure X.
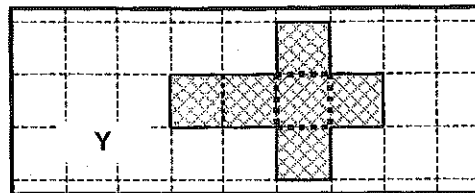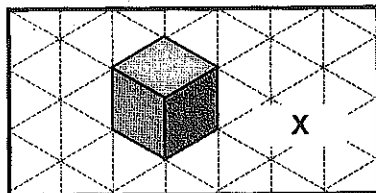
(a) Match the following solid structures to their corresponding jackets.
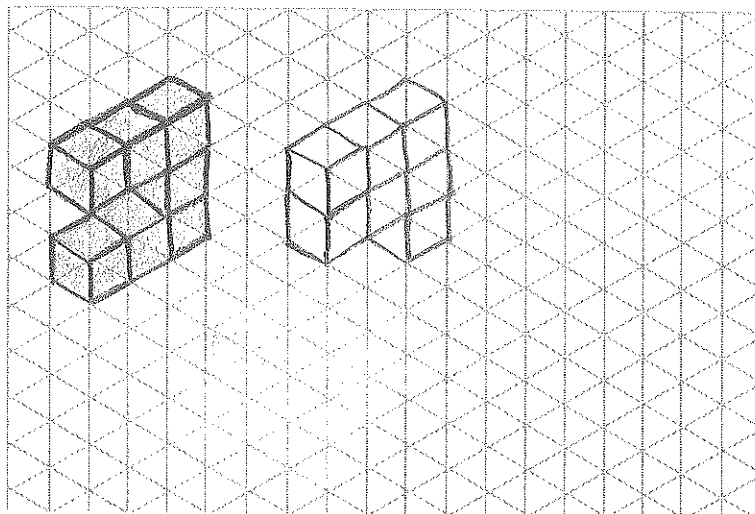

1. D

2. B

3. C

4. A

A. 4

B. 2

C. 3

D. 1

(b) Draw the jackets for the corresponding solids. (Hint: Given below are the "foot-
prints." First, draw the front, right, left and back faces attached to the bottom.
Then, draw the top surface.)
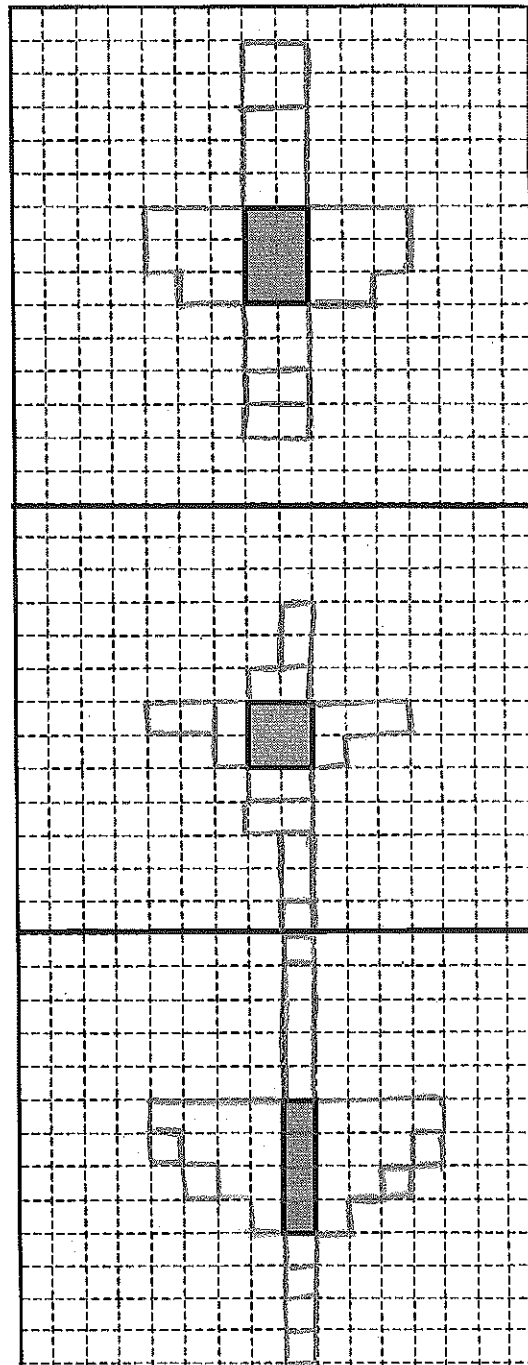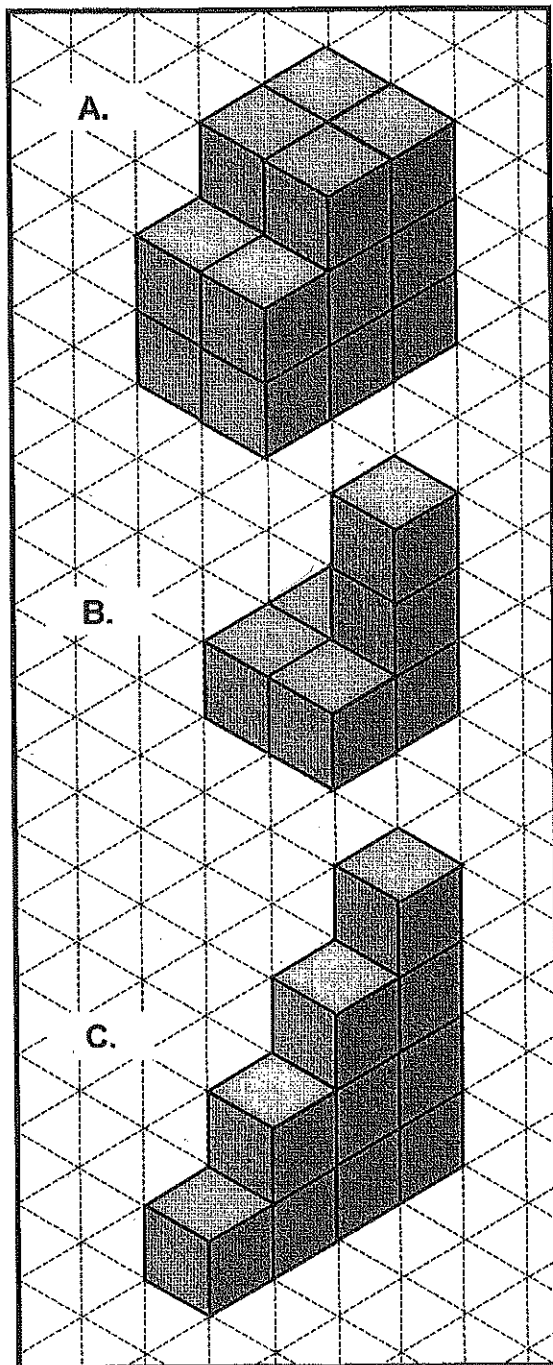
2. Given below are top views with the heights of each section. Some problems also include the side view and front view of the solid. Draw all possible solid structures which have the given projections on isometric paper. Note that not every block needs a supporting block below it).

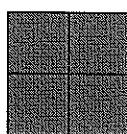| 2	2	3

(a)   Top view with heights

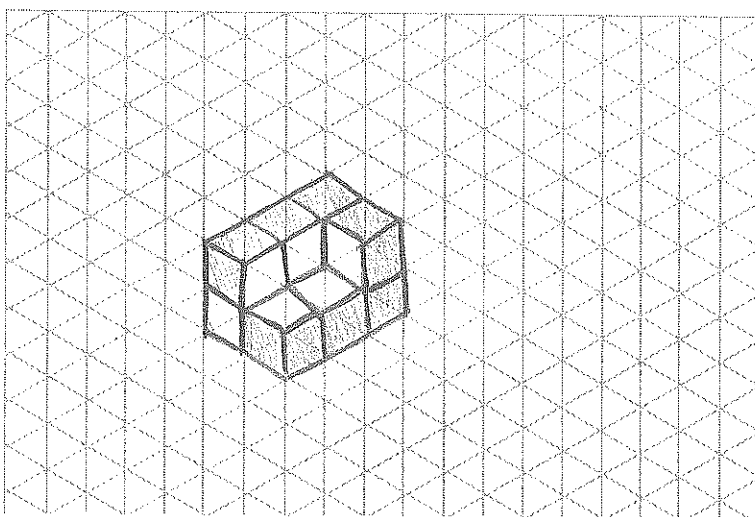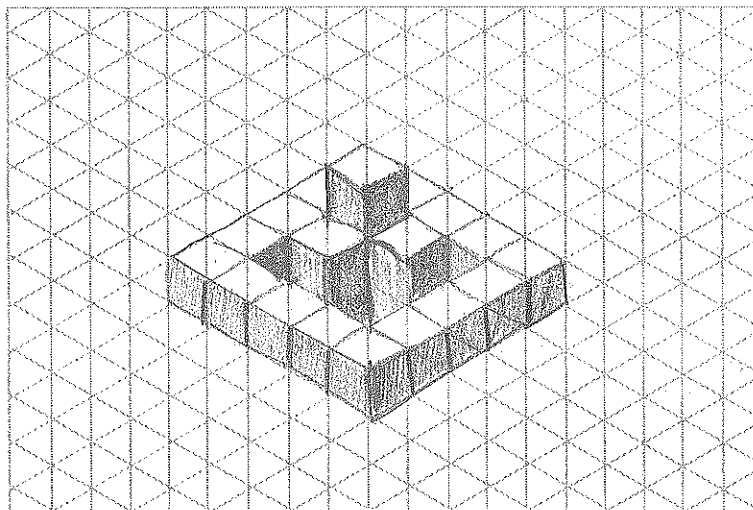| 2	2
2	1
2	1

(b)   Top view with heights   Side view   Front view

same as (b)

(c)

2	2
2	1
2	1

Top view with heights

Side view

Front view

(d)

1	1	1	1	1
1	0	2	0	1
1	2	3	2	1
1	0	2	0	1
1	1	1	1	1

Top view with heights
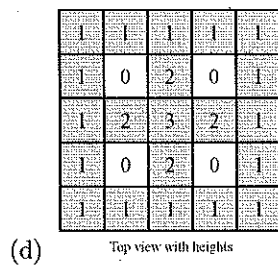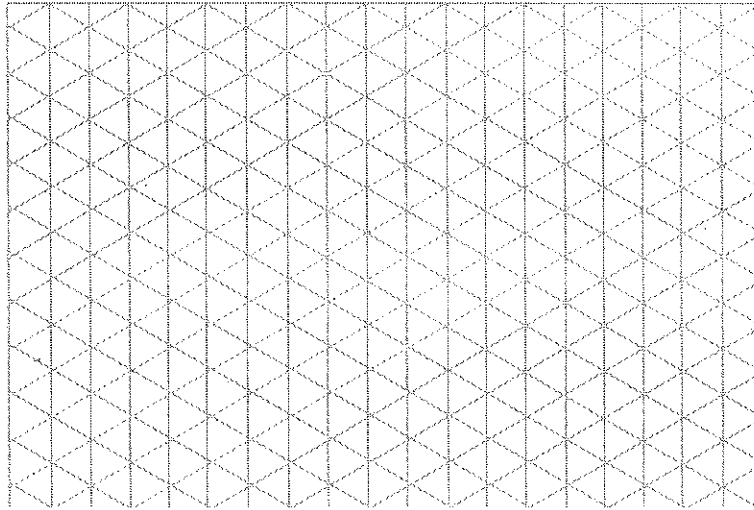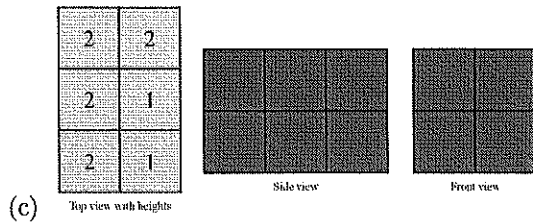
3. Three solid structures are given below. For each, draw on isometric paper the complimentary solid structure that can be joined with the given structure to form a cube. (Hint: First, figure out the dimensions of the smallest cube that works.)