

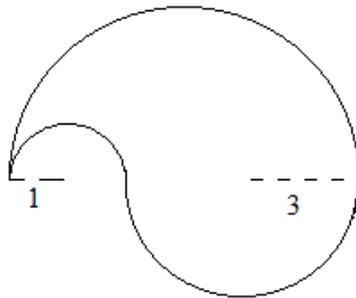
MATH KANGAROO PROBLEMS*

BEGINNERS 04/12/2015

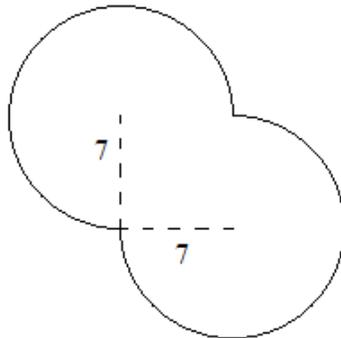
Warm-Up.

(1) Compute the area and perimeter of the following shapes:

(a) P= A=



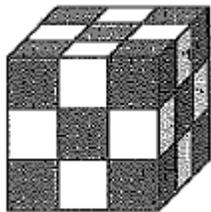
(b) P= A=



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*) The following problems were taken from the 2015 Math Kangaroo Exam Levels 3 and 4, and Levels 5 and 6.

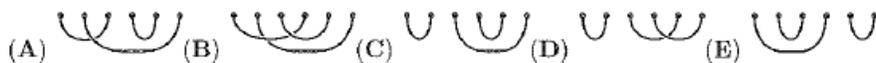
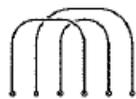
- (1) Jack built a cube using 27 small cubes which are colored either gray or white (see figure). No two of the small cubes which are the same color have a common face. How many white cubes did Jack use?



- (2) Peter has ten balls, numbered from 0 to 9. He gave four of the balls to George and three to Ann. Then each of the three friends multiplied the numbers on their balls. As the result, Peter got 0, George got 72, and Ann got 90. What is the sum of the numbers on the balls that Peter kept for himself?



- (3) Three ropes are laid down on the floor as shown below. You can make one big, complete rope by adding one of the sets of rope ends shown in the pictures below (without changing their positions).
- (a) Which of the sets will make one complete rope?



- (b) Can you come up with another solution?

- (4) We have three transparent sheets with the patterns shown below. We can rotate the three sheets, but not turn them over. Then we put them all one exactly on top of another. What is the maximum number of black squares seen in the square obtained in this way if we look at it from above?



- (5) Anna, Berta, Charlie, David, and Elisa were baking cookies on Friday and Saturday. Over the two days, Anna made 24 cookies, Berta 25, Charlie 26, David 27, and Elisa 28. Over the two days, one of them made twice as many cookies as on Friday, one 3 times as many, one 4 times as many, one 5 times as many, and one 6 times as many. Who baked the most cookies on Friday?
- (6) Grandma bought some candy. She gave each of her grandchildren 4 pieces of candy, and had 2 pieces left. If she wanted to give each of them 5 pieces of candy, she would be 2 candy short. How many grandchildren does she have?

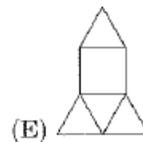
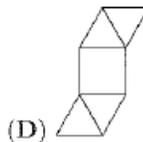
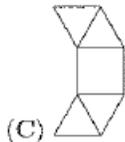
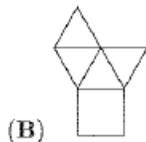
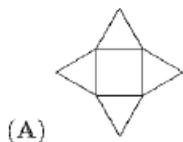
- (7) A student wrote down a natural number. When she divided the number by 9, the remainder was 7. What is the remainder when twice that number is divided by 9?

- (8) The Beginner's Math Circle class plays dominoes. Emmanuelle bought pencils as prizes. She can either give the top three pairs three pencils per winner and give all the other students one pencil, or she can give the top five pairs three pencils per winner and keep two pencils for herself.

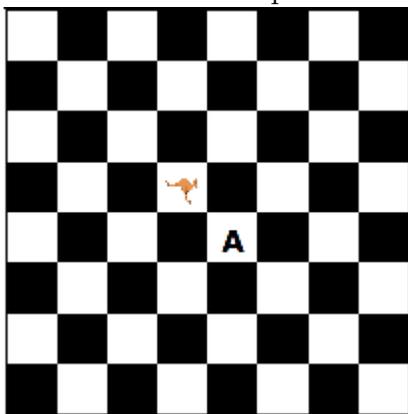
(a) How many pencils did Emmanuelle buy?

(b) How many students are in the Beginners' Math Circle?

- (9) Which of these five nets cannot be the net of a pyramid? [Hint: A net is a flat shape that can be folded up to form a three dimensional object.] Explain.



- (10) Lucy and her mother were both born in February. On March 19, 2015, Lucy adds the year of her birth, the year of her mother's birth, her age, and her mother's age. What result does she get?
- (11) In a bag there are 3 green apples, 5 yellow apples, 7 green pears and 2 yellow pears. Simon randomly takes fruit out of the bag one by one. How many pieces of fruit must he take out in order to be sure that he has at least one apple and one pear of the same color?
- (12) A new chess piece called "kangaroo" has been introduced. In each move, it jumps either 3 squares vertically and 1 square horizontally, or 3 squares horizontally and 1 square vertically, as shown in the picture. What is the minimum number of moves the kangaroo needs to make in order to get from its current position to the square marked with A? Explain.

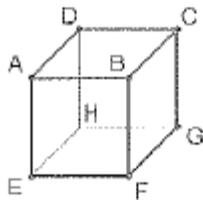


- (13) In this sum, the same letters represent the same digits, and different letters represent different digits. Which digit does the letter X represent?

$$\begin{array}{r}
 X \\
 + \quad X \\
 + \quad Y \quad Y \\
 \hline
 \overline{Z} \quad \overline{Z} \quad \overline{Z}
 \end{array}$$

- (14) In a 4-digit number \overline{abcd} , the digits $a < b$, $b < c$, and $c < d$. What is the largest possible difference $\overline{bd} - \overline{ac}$ for 2-digit numbers \overline{bd} and \overline{ac} ?

- (15) Mary wrote a number on each face of a cube. Then, for each vertex, she added the numbers on the three faces on which that vertex lies (for example, for vertex B she adds the numbers on faces BCDA, BAEF, and BFGC). She obtained 8 numbers. The numbers obtained by Mary for vertices C, D and E are 14, 16 and 24, respectively. What number did she obtain for vertex F?



(16) Four points lie on a line. The distances between them are, in increasing order: 2, 3, k , 11, 12, 14. What is the value of k ? Sketch the line with the four points and the different distances between them.

(17) A bicycle odometer keeps track of the distance ridden by counting the number of times the tire turns. Eric's bike has 16" diameter tires, but he accidentally sets the odometer to 18" diameter tires. After a very long ride, the odometer says Eric has traveled 47 miles. How far has he actually traveled?

(a) Write a formula for the distance ridden (D), in terms of diameter (d) and number of turns (n).

(b) If the tires have a $d=16$ and $D=47$. How many times did the tire turn?

(c) What distance did Eric actually travel?

- (18) The *Reuleaux triangle*, seen below, is a shape which is not technically a triangle at all. It is formed as the intersection of three disks each of radius r , centered at the three vertices of an equilateral triangle of side length r . What is the perimeter of the Reuleaux triangle for a given r ?

