

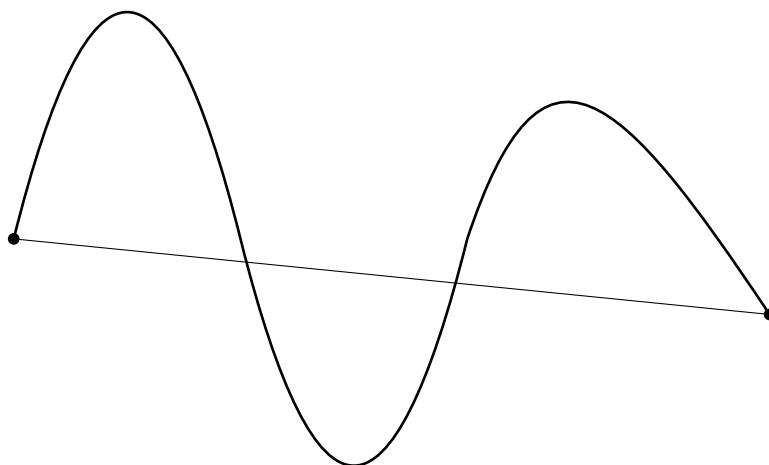
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How to estimate π

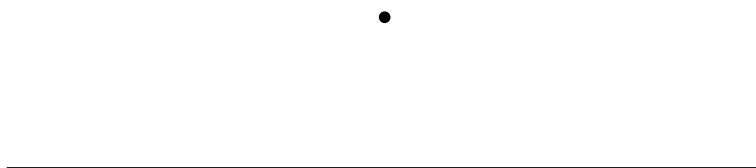
The following sequence of five problems shows how.

Problem 1 *Formulate the triangle inequality and explain why it works.*

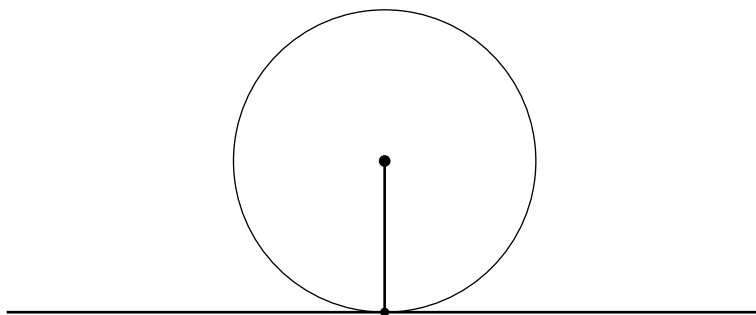
Problem 2 *Prove that the shortest path between two points in the Euclidean plane is the segment of the straight line connecting them.*



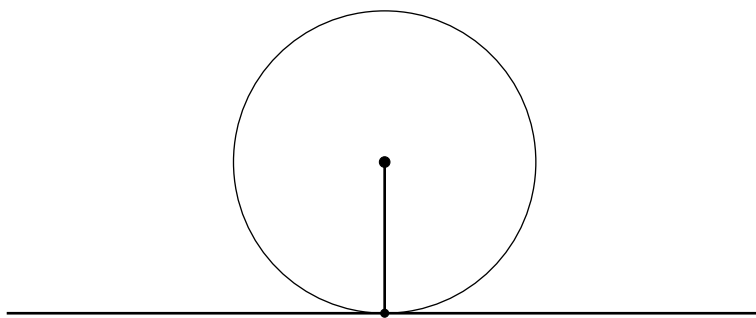
Problem 3 *Prove that the shortest path from a point to a straight line in the Euclidean plane is the perpendicular from the point to the line.*



A straight line is called *tangent* to a circle, if they intersect at one point.



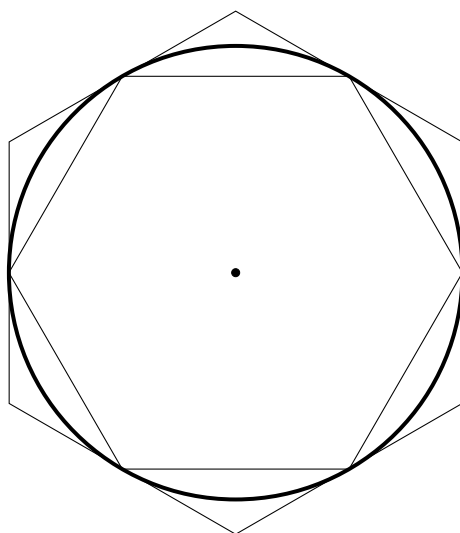
Theorem 1 *A line tangent to a circle is orthogonal to the radius drawn from their common point to the circle's center.*



Problem 4 *Prove Theorem 1.*

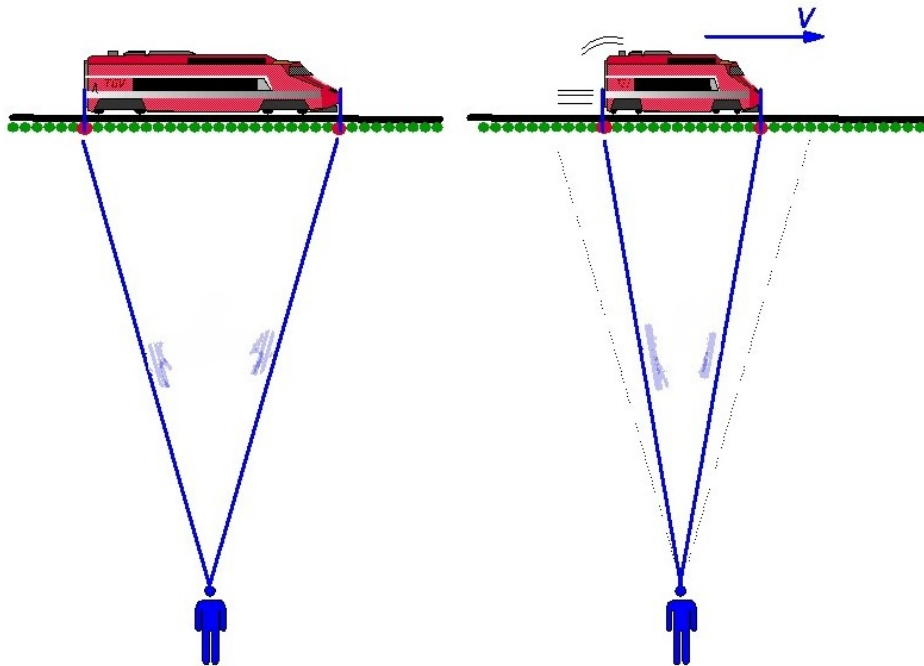
The numbers a and b are called the *lower* and *upper bound* for the number x if $a \leq x \leq b$.

Problem 5 Find the perimeters of regular hexagons inscribed in a circle of radius r and circumscribed around it. Use the perimeters to get the upper and lower bounds for π .



π and geometry of the world

When an object is moving at the speed comparable to the speed of light, the length of the object seems to shorten to a stationary observer, the effect known as *Lorentz contraction*, after a great Dutch physicist [Hendrik Antoon Lorentz](#).



Lorentz contraction

The shortening is described by the following formula.

$$l = l_0 \sqrt{1 - \frac{v^2}{c^2}} \quad (1)$$

Here l_0 is the length of the stationary object, v is the speed of its motion, and $c = 299,792,458 \text{ m/s}$ is the speed of light in the

vacuum.

The Lorentz contraction 1 as well as the *time dilation*

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (2)$$

form the mathematical bedrock of the special relativity theory.



Einstein and Lorentz

Problem 6 *How fast, relative to the speed of light, should an object move to appear shortened twice to a stationary observer?*

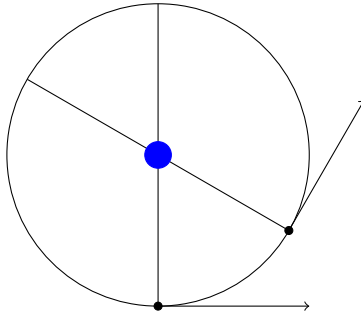
Problem 7 *An expedition is sent to the star system nearest to our Sun, Alpha Centauri, 4.37 light years away. The ship moves at 80% the speed of light (in the vacuum). The expedition spends a year studying the system (in addition to the time of the travel) and then comes back to Earth. How much time would it take them in total? How much time on Earth would pass from the departure of the expedition to its comeback?*

USS Enterprise orbits the Earth in a (near) circular orbit moving at 80% the speed of light.



USS Enterprise

According to Theorem 1, the direction of the motion of the spaceship is perpendicular to the diameter of the orbit passing through the vessel at any moment of time.

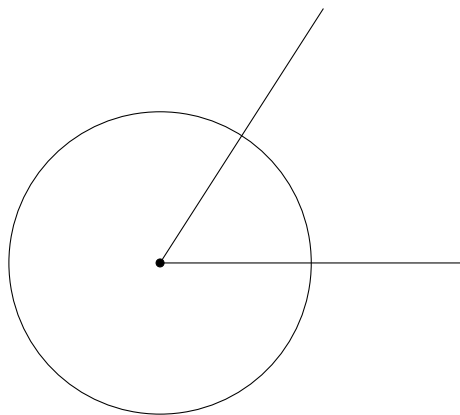


For an observer on Earth, the ratio of the circumference of the orbit to the diameter equals to π . Since the Lorentz contraction

occurs in the direction of the motion, it does not affect the measurement of the orbit's diameter made by the people on board the ship.

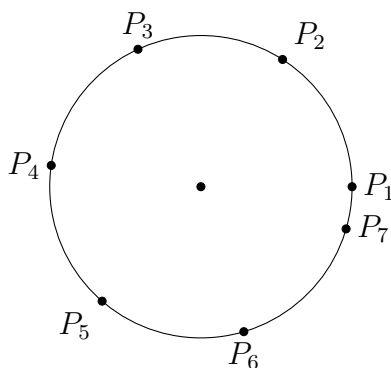
Problem 8 *Find the ratio of the orbit's circumference to the diameter as measured by the ship's navigator.*

One radian is defined as the size of an angle such that if you draw a circle centered at the angle's vertex, the angle subtends the arc equal in length to the circle's radius.



The following problem was suggested by an Intermediate I student, Cooper Komatsu.

Problem 9 *Take a point at the circle and start making one-radius-long steps along the circumference. Would you ever step into the same point twice?*



Problem 10 *Prove that no matter how short an arc of the circumference you take, it will contain a point of the walk suggested by Cooper.*

Useful Fact

The rational numbers

$$\frac{22}{7} = 3.142\dots \quad \text{and} \quad \frac{355}{113} = 3.1415929\dots$$

provide the most accurate approximations of

$$\pi = 3.1415926\dots$$

among the fractions with one- and three-digit denominators respectively.