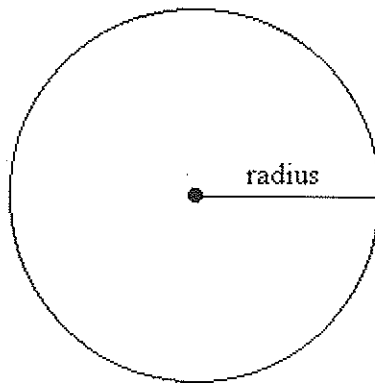


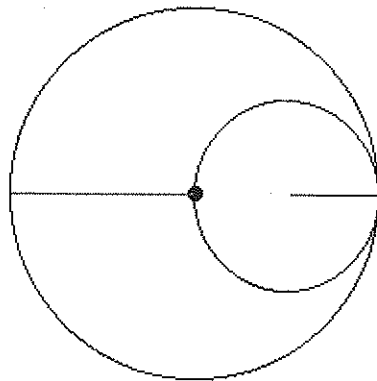
CELEBRATING π DAY

BEGINNERS 03/15/2015

The circumference is the distance around a circle. A circle's circumference is proportional to its radius: Circumference \sim Radius

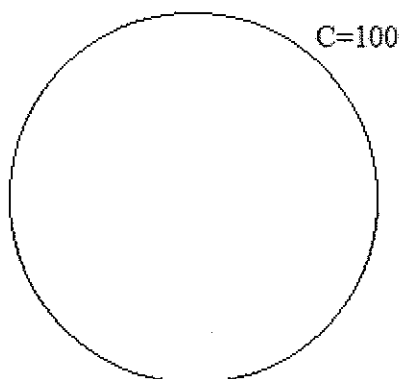


- (1) For some radius r , the circumference of a circle is 20 feet. What is the circumference of a circle when the radius is twice as big?



40 feet

- (2) The circle below has a circumference of 100. How many times bigger is the radius of a circle whose circumference is 300?



3 times

Mathematicians noticed that the ratio between the circle's circumference and the diameter is constant, so that the circumference is roughly equal to three times the diameter, or six times the radius. The Greek letter π , the first letter in the Greek word for "perimeter," is used to denote this mathematical constant.

$$\pi = \text{Circumference} / \text{Diameter}$$

$$\text{Circumference} = \pi \cdot \text{diameter}$$

$$\text{Circumference} = 2\pi \cdot \text{radius}$$

- (1) Solve the following:

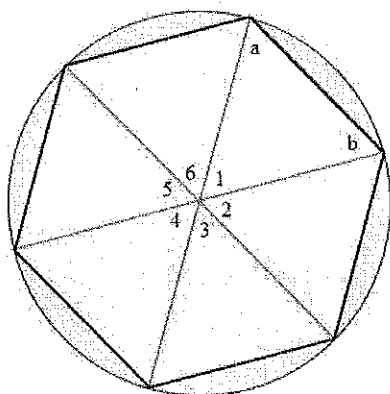
(a) Radius = 3, Circumference = 6π

(b) Radius = 5, Circumference = 10π

(c) Circumference = 6π , Radius = 3 Diameter = 6

(d) Circumference = 10, Radius = $\frac{5}{\pi}$ Diameter = $\frac{10}{\pi}$

How big is π ?



The figure above depicts a regular hexagon inscribed in a circle with radius r . Consider the following:

- A full circle is 360°
- Each of the angles labeled 1-6 is $\frac{360^\circ}{6} = 60^\circ$
- Each triangle is isosceles. Explain.

Because the radii are equal in length.

So, the triangles all have two sides equal in length.

- Angles a and b are equivalent. Explain.

Because the figure is symmetric.

The angles opposite to equal sides are equal.

- Angle $a = \frac{(180^\circ - 60^\circ)}{2} = 60^\circ$
- Each triangle is equilateral.

What is the circumference of the above circle?

$$2\pi r$$

What is the perimeter of the above hexagon?

$$6r$$

Circumference $>$ Perimeter of the hexagon.

Thus, $2\pi > 6$ and $\pi > 3$.

- (1) Cavemen used $\pi = 3$. Ancient Greeks used $\pi = \frac{22}{7}$. For a circle of radius $r = 7$ meters, what is the difference between the circumferences computed by the Ancient Greeks and the cavemen.

(a) What is the circumference of the circle calculated by the cavemen?

$$42 = 2 \times 3 \times 7$$

(b) What is the circumference of the circle calculated by the Ancient Greeks?

$$2 \times \frac{22}{7} \times 7 = 44$$

(c) What is the difference between the two calculations?

$$2$$

(d) Approximate the percentage difference.

$$\frac{44-42}{42} \times 100 = \frac{2}{21} \times 100 = \text{approximately } 9.5\%$$

- (2) Cavemen measured the radius of a circle and computed the circumference to be 84 cm. What circumference would the Ancient Greeks have calculated?

Cavemen: $84 = 2 \times 3 \times r$
 $r = \frac{84}{6} = 14 \text{ cm}$

Ancient Greeks: $2 \times 14 \times \frac{22}{7} = 88 \text{ cm}$

Later, π was computed more precisely.

$$\pi = 3.1415926535\dots$$

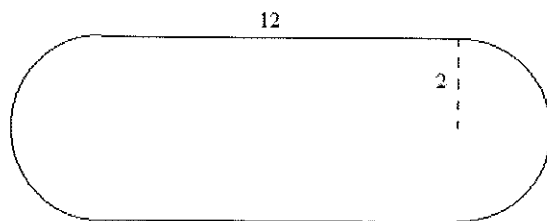
You can remember the digits using the a sentence mnemonic, a phrase in which the number of letters of each successive word corresponds to a digit of pi:

"Wow, I made a great discovery!" (3.14159)

$$3.14159 = 3 + 14159/100,000$$

For many practical purposes 3.14 is enough. In computations, keep as π (unless asked for a numerical value).

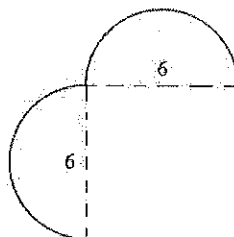
(1) Compute the perimeters of the following shapes:



$$12 + 12 + 2 \times \pi \times 2$$

$$= 24 + 4\pi$$

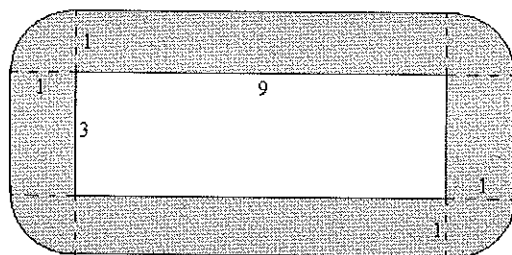
(a)



$$6 \times 2 + 2 \times \pi \times 3$$

$$= 12 + 6\pi$$

(b)

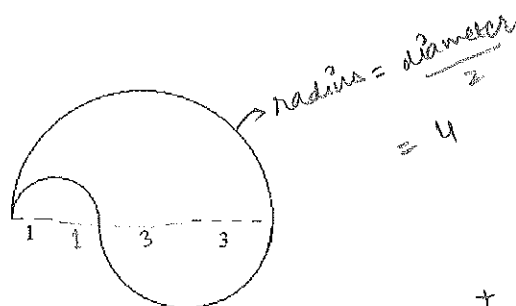


$$9 \times 2 + 3 \times 2 + 2 \times \pi \times 1$$

$$= 18 + 6 + 2\pi$$

$$= 24 + 2\pi$$

(c)



(d)

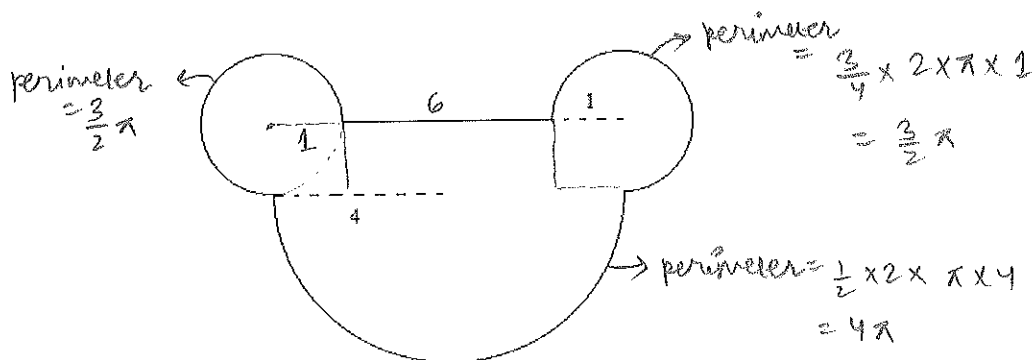
$$\frac{1}{2} \times 2 \times \pi \times 1$$

$$+ \frac{1}{2} \times 2 \times \pi \times 1$$

$$+ \frac{1}{2} \times 2 \times \pi \times 3$$

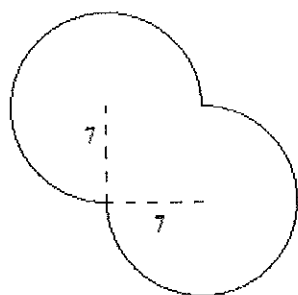
$$= 4\pi + \pi + 3\pi$$

$$= 8\pi$$

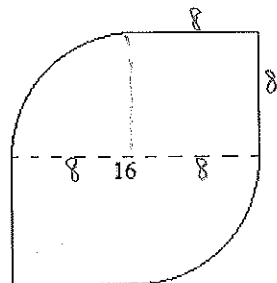


(e)

So, total perimeter $= \frac{3}{2}\pi + \frac{3}{2}\pi + 4\pi + 6 = 7\pi + 6$



(f)



(g)

- (2) A vine starts at the bottom of a 28-foot tree trunk and wraps around to the top, going around exactly 7 times. If the diameter of the trunk is 3 feet, what is the length of the vine?

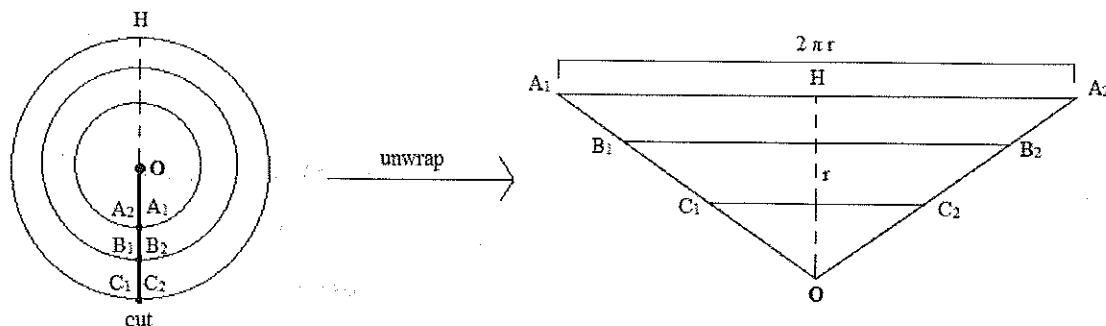
Circumference of the trunk $= 3\pi$

The vine goes around 7 times

So, it covers $7 \times 3\pi$ feet

$= 21\pi$ feet

To derive the formula for the area of a circle, we use the diagram below.



We first draw three concentric circles, the largest of which has a radius of r . Imagine cutting through the bold segment and “unwrapping” the circle so the edges are flattened out. This results in the isosceles triangle above. The height of the triangle, OH , is the radius r , and the distance between A_1 and A_2 is the unfolded circle and, thus, equals the circumference $2\pi r$. The area of the triangle, which is also the area of the circle with radius r is:

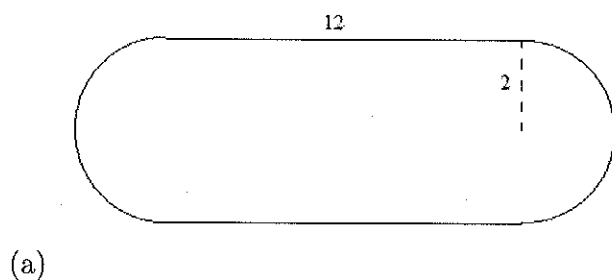
Area of circle = Area of triangle

$$A = \frac{1}{2} \cdot \text{base} \cdot \text{height}$$

$$A = \frac{1}{2} \cdot 2\pi r \cdot r$$

$$A = \pi r^2$$

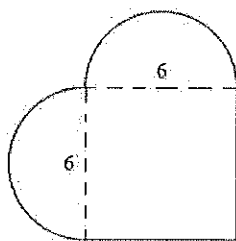
(1) Compute the areas of the following shapes:



Area of rectangle + area of circle

$$12 \times 4 + \pi \times 2^2$$

$$= 48 + 4\pi$$

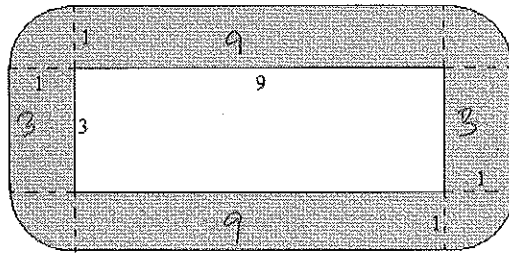


Area of square + Area of circle

$$6^2 + \pi \times 3^2$$

$$= 36 + 9\pi$$

Area of shaded region

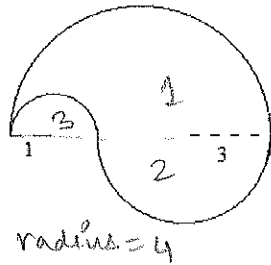


Area of the four rectangles +
Area of circle
- Area of rectangle inside

$$= 2 \times 9 \times 1 + 2 \times 1 \times 3 + \pi \times 1$$

$$= 24 + \pi$$

(c)



radius = 4

Area of semicircle 1 + Area of semicircle 2
- Area of semicircle 3

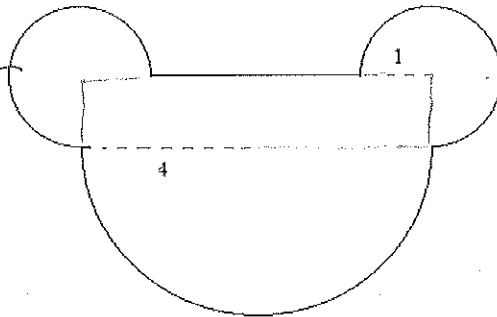
$$= \frac{\pi \times 4^2}{2} + \frac{\pi \times 3^2}{2} - \frac{\pi \times 1^2}{2}$$

$$= \frac{16\pi}{2} + \frac{9\pi}{2} - \frac{\pi}{2} = \frac{24\pi}{2} = 12\pi$$

(d)

$\frac{3}{4}$ of an entire circle

$$\text{Area} = \frac{3}{4} \times \pi r^2$$

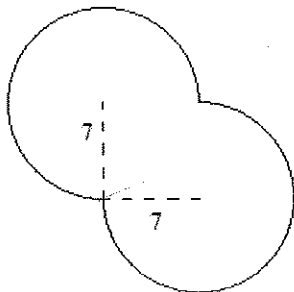


$$2 \times \frac{3}{4} \times \pi \times 1^2 + 8 \times 1 + \frac{1}{2} \times \pi \times 4^2$$

$$= \frac{3}{2} \pi + 8 + 8\pi$$

$$= \frac{19}{2} \pi + 8$$

(e)

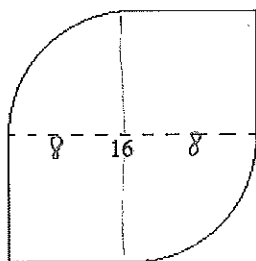


$$2 \times \frac{3}{4} \times \pi \times 7^2$$

$$= \frac{147}{2} \pi$$

(f)

$$\frac{49}{3}$$



$$2 \times 8 \times 8 + 2 \times \frac{1}{4} \times \pi \times 8^2$$
$$= 128 + 32\pi$$

(g)

