LAMC Intermediate I & II **February 15, 2015**

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Math Kangaroo Preparation Session

Problem 1 *In a magic square, the sum of the numbers in each row, column, and along the diagonals is the same. Fill out the magic square below.*

16	3	
	10	

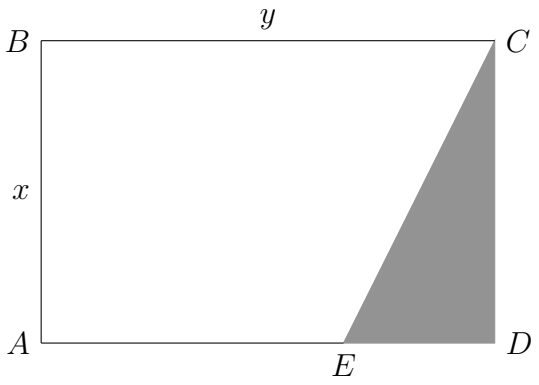
Problem 2 *A melon weighs 4/5 kg more than 4/5 of the same melon. How much does the melon weigh?*

Problem 3 *Let x be a five-digit decimal integer. Let y be the number obtained from x by reversing the order of the digits. Prove that the difference $x - y$ is divisible by 99.*

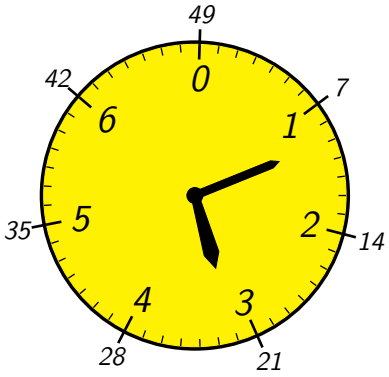
Problem 4 *Let x be a five-digit binary integer. Let y be the number obtained from x by reversing the order of the digits. Prove that the difference $x - y$ is divisible by 3.*

Problem 5 *Ten married couples take part in an experiment. How many ways are there to choose a group of seven participants so that there are no married couples in the group?*

Problem 6 *The area of the triangle CDE is one sixth of the area of the rectangle $ABCD$. $|AB| = x$, $|BC| = y$. Find the perimeter of the triangle.*



Problem 7 *The planet of Heptadium in a galaxy far, far away makes one full rotation around its axis in 7 heptahours. The folks inhabiting Heptadium have heptahour clocks similar to the one pictured below.*



They further divide a heptahour into 49 heptaminutes and a heptaminute into 49 heptaseconds. The heptahours are marked on the inside of the dial, the heptaminutes – on the outside.

At noon, the hour and minute hands of a heptaclock point in the same direction (at zero). What is the nearest moment of time, in heptahours, when the hands will point in the same direction again?

Problem 8 *Three kids, Alice, Bob, and Charlie, make the following statements.*

Alice: The distance between Bob and myself is greater than twice the distance between myself and Charlie.

Bob: The distance between Charlie and myself is greater than twice the distance between myself and Alice.

Charlie: The distance between Bob and myself is greater than twice the distance between myself and Alice.

At least two of the kids are telling the truth. Which one of them, if any, tells a lie?

Problem 9 *What is the last digit of the number* 3^{2015}*?*

Problem 10 *The British mathematician Augustus De Morgan was* x *years old in the year* x^2*. He died in 1871. What year was he born?*

Recall that De Morgan's laws are the following.

$$\neg(A + B) = \neg A \times \neg B$$

$$\neg(A \times B) = \neg A + \neg B$$

Problem 11 *Prove De Morgan's laws.*

Problem 12 *Given* $\dfrac{1}{1+\dfrac{1}{x}} = 5$, *find the value of the following continued fraction.*

$$\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+x}}}}$$

Problem 13 *An infinite sequence of numbers begins with* $1, -1$. *Further on, each member of the sequence equals to the product of the previous two. Find the sum of the first 2015 sequence members.*

Problem 14 *On an island of knights and liars, knights always tell the truth, liars always lie. The following conversation takes place on the island.*

First person: There are no more than three of us in the room. Each of us is a liar.

Second person: There are five people in the room. Three of us are liars.

Third person: There are at most four people in the room. Not every one of us is a liar.

How many people are there in the room and how many of them are liars?

Problem 15 *Prove that an angular bisector is the set of all the points in the plane equidistant from the sides of the angle. (Note that you need to prove two statements, not one.)*

Problem 16 *The segment AD is the angular bisector of the angle BAC in the triangle ABC. The area of the triangle ABD equals the area of the triangle ACD. Prove that the triangle ABC is isosceles.*

Problem 17 *A tanker truck delivered gas to three different gas stations. At the first one, 30% of the gas was pumped out of the tanker. At the second station, the truck pumped out 40% of the remaining fuel. At the third station, the truck unloaded 50% of what was left. What percentage of the original amount of gas was left in the tanker?*

Problem 18 *Sides AB and AC of the triangle ABC are diameters of two circles that intersect at points A and D. Prove that D is the base of the altitude dropped from the vertex A on the side BC.*

Problem 19 *In a certain class, 1/8 of the students received a C for a math test, 1/6 received a B, and 2/3 received an A. Except for F, there were no other grades for the test. How many students received an F, if there were less than 30 students in the class?*

Problem 20 *Prove that the numbers* 16, 1156, 111556, 11115556, 1111155556 ... *are all perfect squares.*