

Counting and probabilities

January 25, 2015

1. You have 3 distinct pens, which you want to distribute to 5 people. It is possible that a person gets 2 or all 3 pens. In how many ways can this happen? (there is a very short method, and a longer one)

2. In a lake, there are 7 lily pads. A frog sitting in the water decides to jump on all of them, and wants to jump on one of them exactly twice, but not consecutively. In how many ways can this happen?

3. You have 10 rings. Three of them, made of gold, look identical, while the other 7 are distinct. You decide to put 5 of them on your right hand (one on each finger), and then take a picture. How many pictures can be created?

4. Prove that the number of ways can you climb n stairs, in jumps of 1 step or 2 steps, is $\frac{1}{\sqrt{5}} \left(\frac{1+\sqrt{5}}{2} \right)^{n+1} - \frac{1}{\sqrt{5}} \left(\frac{1-\sqrt{5}}{2} \right)^{n+1}$

5. A point (x,y) is randomly chosen from inside the rectangle with vertices $(0,0)$, $(4,0)$, $(4,1)$ and $(0,1)$. What is the probability that $x < y$?

6. A coin is altered so that the probability that it lands on heads is less than $\frac{1}{2}$ and when the coin is flipped four times, the probability of an equal number of heads and tails is $\frac{1}{6}$. What is the probability that the coin lands on heads?

7. Suppose you have a floor with parallel lines, with distance 1 between them. You have a needle of length 1, which you throw on the floor. Try to express the probability that the needle intersect a line as the area of a region in the plane.

8. A frog sits on the point 0 on the real line, and can jump in steps of 1 or 2, either to the left or to the right. In how many ways can he get to the point 4, using 12 jumps? In how many ways can he get to the point 7, using 16 jumps?

9. Point C is chosen at random atop a 5 foot by 5 foot square table. A circular disk with a radius of 1 foot is placed on the table with its center directly on point C. What is the probability that the entire disk is on top of the table (i.e. that none of the disk hangs over an edge of the table)

10. In a group of 10 people, we have two identical twins (and identically dressed) and 3 identical triplets (identically dressed). You chose 8 persons, you line them up and take a picture. How many pictures are possible?