

Oleg Gleizer
prof1140g@math.ucla.edu

Warm-up

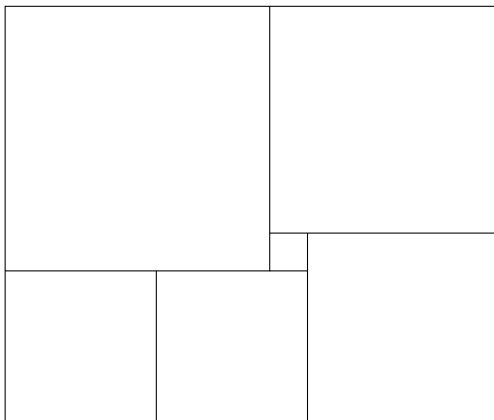
Problem 1 *There are 24 lbs of sand in a bag. Given a balance scale without weights and a bunch of empty bags, fill one of the extra bags with 9 lbs of sand.*

Problem 2 *The number of the students absent in a class equals to one sixth of the number of the students in presence. If one student leaves, the number of the absent students will become one fifth of the numbers of the students present. How many students are there in the class?*

Problem 3 Find the value of the following expression. Do not use a calculator!

$$\frac{\left(13\frac{1}{4} - 2\frac{5}{27} - 10\frac{5}{6}\right) \times 230\frac{1}{25} + 46\frac{3}{4}}{\left(1\frac{3}{7} + \frac{10}{3}\right) \div \left(12\frac{1}{3} - 14\frac{2}{7}\right)} =$$

Problem 4 The rectangle below is built of squares. Find the side length of the largest square, if the side length of the smallest square equals one.



Problem 5 Find a natural number x such that three of the five inequalities below are correct while two are not.

1. $2x > 70$

2. $x < 100$

3. $3x > 25$

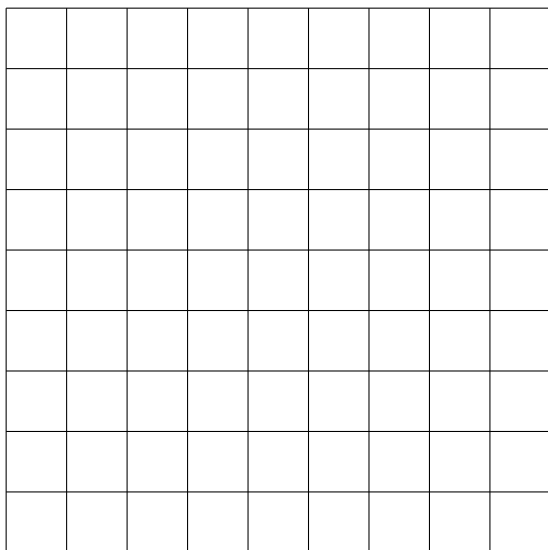
4. $x > 10$

5. $x > 5$

Problem 6 Twenty different points are marked on a sheet of paper. A straight line is drawn through every two points. What is the least and what is the greatest number of the lines possible?

Problem 7 *A pile of consecutively numbered pages has fallen out of a folder. The first page of the pile is numbered 463. The number of the last page is made of the same digits, but in a different order. How many (two-page) sheets of paper were dropped?*

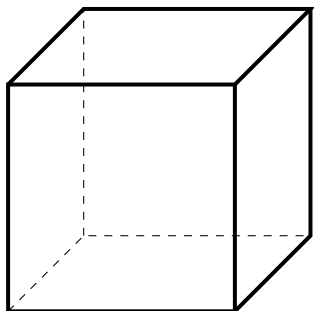
Problem 8 *Two players play on a 9×9 grid. They take turns moving any number of steps either right or up, starting from the lower-left corner. The winner is the last to make a move. Find the winning strategy for the game.*



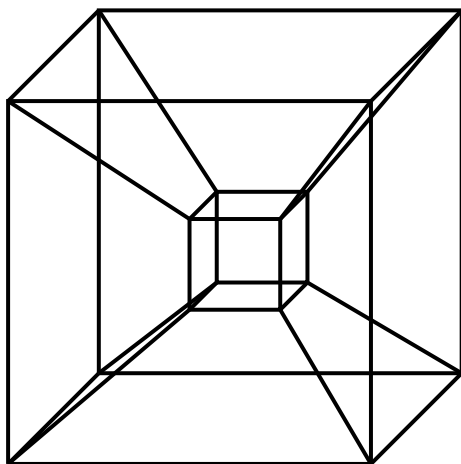
Problem 9 *Find ten different natural numbers such that the product of any two of them is divisible by the sum of all the ten numbers.*

Problem 10 *A hundred points are marked on a sheet of paper. Is it always possible to draw a straight line that separates the points into two groups of fifty points each?*

Problem 11 Find the surface area of a 3D cube of side length a .

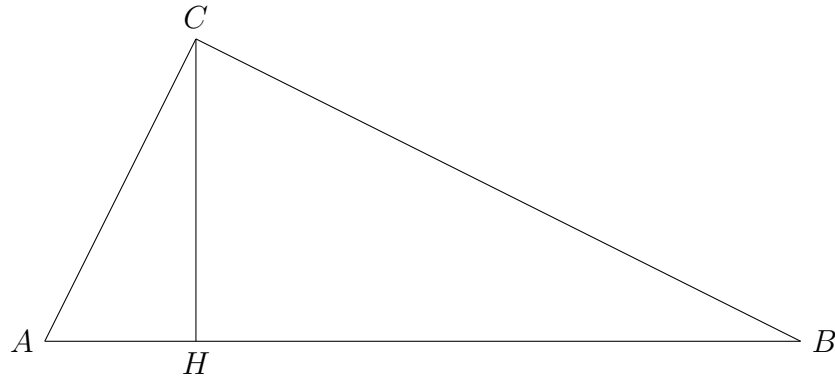


Problem 12 Find the 3D area of a 4D cube of side length a .



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Problem 13 *The angle ACB of the triangle ABC on the picture below is right. Let H be the foundation of the altitude dropped from the vertex C to the side AB .*

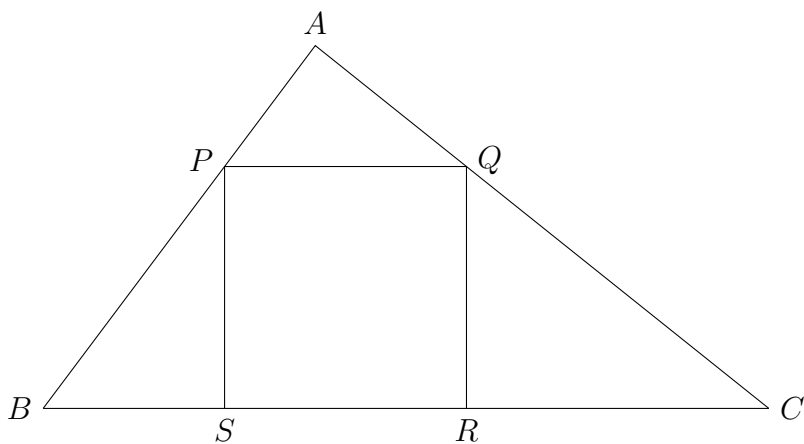


Prove the following two statements.

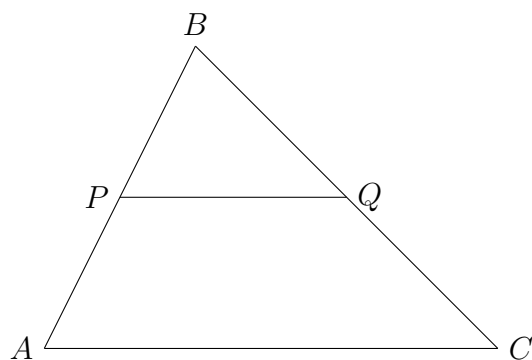
$$|AC|^2 = |AB| \times |AH|$$

$$|CH|^2 = |AH| \times |BH|$$

Problem 14 *The square $PQRS$ is inscribed in the triangle ABC as shown on the picture below. The length of the side BC is a . The side length of the square is s . Find the area of the triangle ABC .*

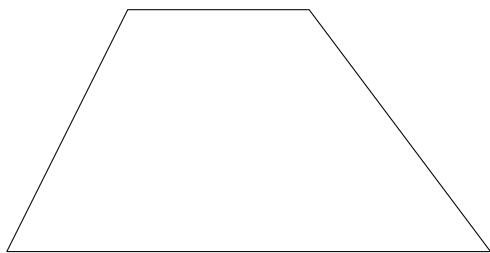


A *midline* in a triangle is a line joining centers of two sides.

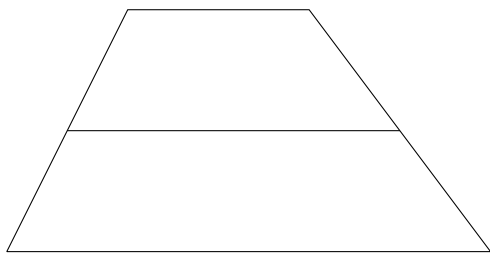


Problem 15 *Prove that the midline PQ is parallel to AC and that $|AC| = 2|PQ|$.*

A *trapezoid* is a quadrilateral having a pair of parallel opposite sides.

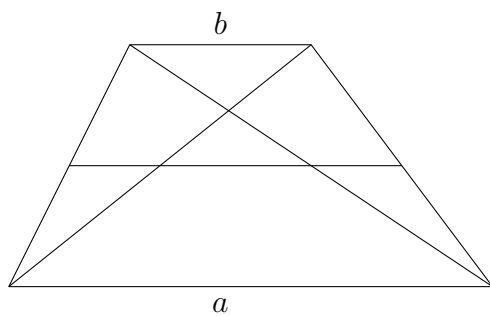


A midline of a trapezoid is the line connecting the centers of the non-parallel sides.



Problem 16 *The trapezoid on the picture above has the lower side of length a and the upper side of length b . What is the length of its midline?*

Problem 17 *Once again, the length of the lower side of the trapezoid is a and the length of its upper side is b . Find the length of the segment cut from the midline by the trapezoid's diagonals.*



Problem 18 *Prove that midpoints of the sides of any quadrilateral are vertices of a parallelogram.*

For what quads would the parallelogram be

- *a rectangle?*
- *a rhombus?*
- *a square?*