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Math Wrangle

The following are the rules and a few comments on them. Please note that some of the rules are different from those of the sample wrangle.

- During the first hour, the teams solve problems in different rooms. Intermediate I meets in the graduate student lounge, MS 6620. Intermediate II meets in their regular room, MS 6201.
- It will help if each team splits into groups, each group tasked with solving a problem or a few. Instructors will be prepared to help splitting the teams into groups if asked for it.
- If a group is finished solving its problems, they should try to help other groups.
- If you can't solve a problem, do not get stuck! Just pass it and try another one.
- If you have time, practice presenting the problems you have solved.

- The teams meet in the graduate student lounge, MS 6620, for the second hour.
- Teams don't have to present the problems in the same order they appear in the handout. A team can challenge the opposing team to solve any unsolved problem at any time!
- The first team to challenge will be chosen at random, possibly by a coin toss or by playing rock-paper-scissors.
- Choosing the right problem for a challenge should be a part of the team's strategy. For example, a team can challenge to a hard problem for which they themselves have a solution. Or you can bluff!
- The challenged team can either present a solution or return the challenge. Each return raises the value of the problem by one point. The fifth return is final.
- It's the job of the opposing team, not of the instructors or the judges, to listen carefully to the other side's presentation trying to find mistakes in the solution!
- The judges will award points without going into long justifications.
- You will have five minutes for presenting a problem and three more for answering questions. The timekeeper will stop your presentation as soon as your time is out. No complains please!

- There may be not enough time to present all the problems. Choose wisely!
- A team that behaves rowdy and/or asks unrelated questions will be deducted one point for each foul.

Problem 1

1 pt

There are four apples in a basket. Is it possible to divide the apples evenly between four people without cutting so that one apple is left in the basket? If you think it is possible, please show how. If you think it's not possible, please explain why.

Problem 2

2 pts

Prove that out of eight integers, one can always choose two such that their difference is divisible by seven.

Problem 3**3 pts**

Is it always possible to choose two integers out of a million so that their sum is divisible by seven? Why or why not?

Problem 4**3 pts**

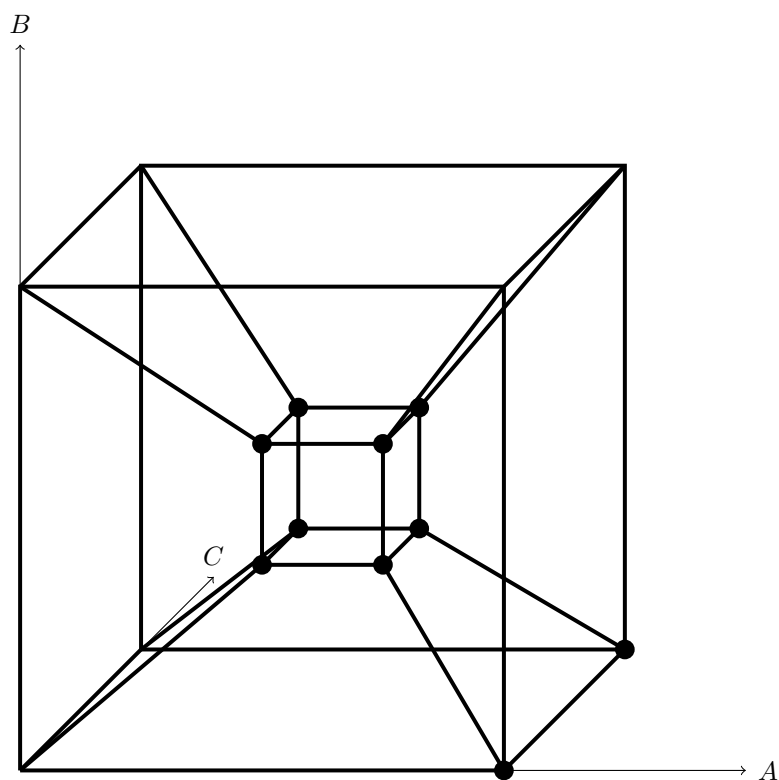
Solve the following equation.

$$2013 + \frac{1}{2 + \frac{1}{3 + \frac{1}{4 - \frac{17}{x}}}} = 2014$$

$$x =$$

Problem 5**3 pts**

Simplify the FDNF expression corresponding to the marked vertices of the following tesseract.



Problem 6**3 pts**

30 coins lie on the table heads up. You are allowed to turn over any 29 coins at a time. Is it possible to turn all the coins tails up? If you think it is possible, please show how. If you think it's not possible, please explain why.

Problem 7**3 pts**

Given the initial set of numbers 1, 2, 3, 4, 5, and 6, it is allowed to add 1 to any pair of the numbers in the set. Can one make all the numbers in the set equal? If you think this is possible, please show how. If you think this is impossible, please explain why.

Problem 8**3 pts**

Josephus and his 72 soldiers were trapped in a cave, with the Romans blocking the exit. Instead of surrendering themselves to the Romans, the Jewish soldiers decided to commit suicide. They formed a circle. Then a fighter killed a neighboring fighter on his left with a blow of his sword and passed the sword further left. The fighter who received the sword did the same. This way, they were killing one another until there was one man left standing. That one was supposed to kill himself. The last one happened to be Josephus who chose to live and wrote down the story. What was the position of Josephus in the circle (relative to the soldier who made the first blow)?

Problem 9**4 pts**

Three boys, Aaron, Baldwin, and Calvin, run a race. Before the race, four spectators make the following predictions.

Aaron will win.

Calvin will finish ahead of Baldwin.

Baldwin will finish right after Aaaron.

Calvin will not win.

An even number of the guesses turns out to be correct. Who got the first, second, and third place?

Problem 10**4 pts**

Prove that logical multiplication is associative.

$$(A \times B) \times C = A \times (B \times C)$$

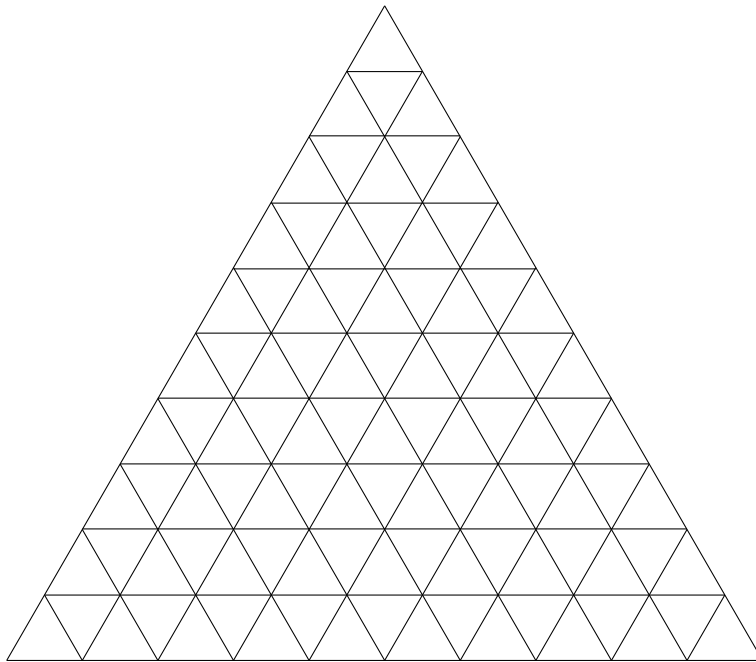
Problem 11**4 pts**

The surface of a soccer ball is made of 32 parts, black pentagons and white hexagons, as seen on the picture below. How many pentagons and hexagons are there?



Problem 12**5 pts**

A triangular building is divided into one hundred triangular rooms as on the picture below. Each room has a door in the center of each of its walls, with the exception for the external walls (the walls that are also the walls of the building). What is the largest numbers of the rooms one can visit without visiting any single room twice? Hint: think chess.



Problem 13**5 pts**

At a particular moment of time in a planetary system far far away, one astronomer on each of the planets looks at the planet that is the nearest to her/his own. At this moment of time, all the distances between the planets are pairwise different. Given that the number of the planets is odd, prove that there is a planet no astronomer is looking at.