LAMC Intermediate I & II **December 7, 2014**

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Math Wrangle Training

Problem 1 **3 pts**

Four girls, Melissa, Jessica, Sandy, and Nicole, were playing in a room. They got a bit wild and someone accidentally broke a wall mirror. When asked who did it, they responded as follows.

Jessica – It wasn't me!

Sandy – It was Nicole!

Melissa – No, it was Sandy!

Nicole – Sandy is a liar.

Only one of the girls said the truth. Who broke the mirror?

Problem 2 **4 pts**

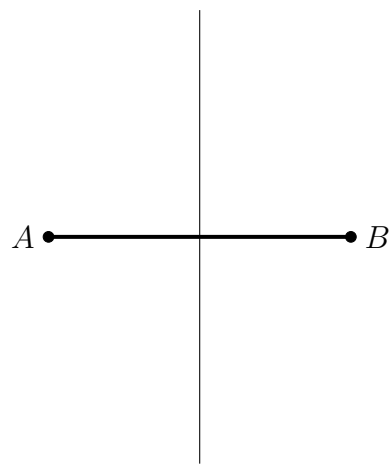
At noon, the hour and minute hand of a watch point in the same direction. When would the hands point in the same direction next time?

Problem 3 **5 pts**

In a herd of 101 cows, each animal weighs a whole numbers of pounds. If any one cow is removed from the herd, the remaining cows can be divided into two herds of equal total weight. Prove that all the cows weigh the same.

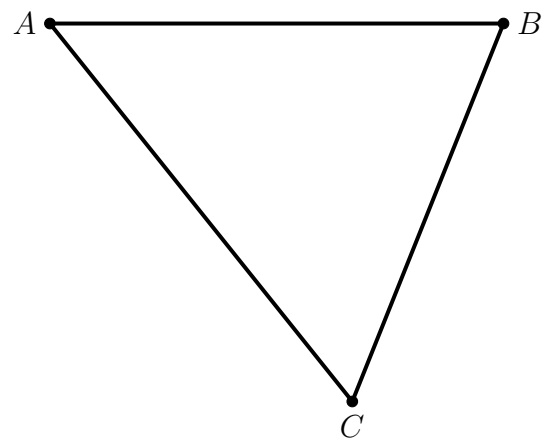
Problem 4 **3 pts**

Prove that a perpendicular bisector of a segment in the Euclidean plane is the set of all the points in the plane equidistant from the endpoints of the segment.



Problem 5 **4 pts**

Prove that three perpendicular bisectors of the sides of any triangle in the Euclidean plane intersect at one point.



Prove that it is possible to circumscribe a circle around any triangle in the Euclidean plane.

Problem 6 **5 pts**

We are given a point light source in 3D emitting light in all directions. Is it possible to choose four opaque balls and place them in the space so that

1. the light source is outside of the balls;

2. the balls do not intersect one another (have no common points); and

3. they completely block the light from the source for a distant observer (i.e. every ray of light emanating form the source meets one of the balls)?

If you think it is possible, please explain how. If you think it's not possible, please explain why.

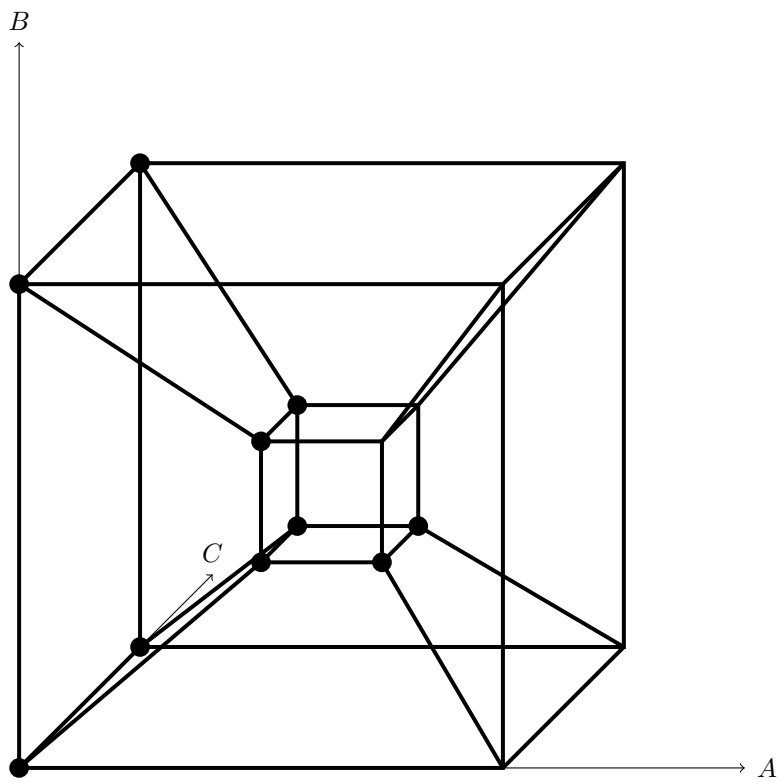
Problem 7 **3 pts**

Prove that the Boolean algebra addition is distributive with respect to multiplication without using the truth table.

$$A + BC = (A + B)(A + C)$$

Problem 8 **4 pts**

Write down the FDNF expression corresponding to the marked vertices of the following tesseract. Then use the geometric approach to simplify.



Problem 9 **5 pts**

Solve the following cryptarithm.

$$
\begin{array}{cccccccc}
 & & & & F & I & V & E \\
 & & \times & & F & I & V & E \\
\hline
 & & & \sqcup & \sqcup & \sqcup & \sqcup & F \\
 & & \sqcup & \sqcup & \sqcup & \sqcup & I & \\
 & \sqcup & \sqcup & \sqcup & \sqcup & V & & \\
\sqcup & \sqcup & \sqcup & \sqcup & E & & & \\
\hline
\sqcup & \sqcup & \sqcup & \sqcup & \sqcup & \sqcup & \sqcup & \sqcup
\end{array}
$$

Problem 10 **3 pts**

A mailman takes mail out of a public mail box five times a day. If he opens the box at equal time intervals starting at 7:00 AM and ending at 7:00 PM, what is the length of each time interval?

Problem 11 **4 pts**

What's more among the positive integers less than one million, the numbers that have no digit seven or the numbers that have it?

Problem 12 **5 pts**

Someone slides a comb having six teeth per centimeter against a stationary comb that has five teeth per centimeter. While sliding, the experimentator looks at a source of light through the combs. For each comb, all the teeth have equal width and so do the gaps between them. At what speed would the lit spaces (the spaces where the light gets through the combs' teeth) be moving, if the sliding comb moves at the speed of 1 centimeter per second?