

The Island of Knights and Liars

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Island of Knights and Liars

There are two types of inhabitants on the *Island of Knights and Liars*:

- *Knights* always tell the truth;
- *Liars* always lie;

Occasionally, *tourists* also visit the island. Tourists sometimes lie and sometimes tell the truth.

1. You ask two people you meet on the Island, Tom and Tim, if they are Knights.
 - Tom says he is a Knight. Can you figure out who Tom is?

 - Tim says he is a Liar. Can you figure out who Tim is?

2. You encounter three Islanders, Mathew, Jack, and Kate. You ask Mathew if he is a Knight.
 - Mathew answers the question, but he mumbles and you can't hear his answer.
 - You ask Jack what Mathew said. Jack says that Mathew said he was a Liar.
 - Kate says, "Don't believe Jack, he is a Liar!"

Who are these Islanders? (Note: they are not tourists.)

3. While visiting the Knights and Liars Island, I had a conversation with a local Knight. I asked him the same question twice, and he gave me two different answers. What could be my question?

4. Andy and Ben live on the *Island of Knights and Liars*. Andy says: “We are both liars”. Who is Andy and who is Ben? (Note: since they live on the island, they are not tourists.)

5. Alice and Bob live on the *Island of Knights and Liars*.

- Alice says: “We are the same kind”;
- Bob says: “We are different kinds”.

Who is Alice and who is Bob?

Cryptarithms are mathematical puzzles in which the digits are replaced by letters of the alphabet. Remember that there are 10 digits: 0, 1, ..., 8, 9. Moreover, each letter represents the same digit throughout the problem.

To solve a cryptarithm means to find what digits correspond to what letters so that you get a valid mathematical equality.

Solve the following cryptarithms. (Note that sometimes several solutions are possible.)

$$1. \begin{array}{r} E \ G \ G \\ + \ E \ G \ G \\ \hline P \ A \ G \ E \end{array}$$

$$E = \quad G = \quad A =$$

$$2. \begin{array}{r} S \ H \ E \\ + \ E \ E \ L \\ \hline E \ L \ S \ E \end{array}$$

$$S = \quad H = \quad E = \quad L =$$

$$3. \begin{array}{r} M \ O \ M \\ + \ M \ O \ P \\ \hline Y \ O \ Y \ O \end{array}$$

$$M = \quad O = \quad Y = \quad P =$$

Fun problems

1. Kara noticed that the sum of her age, her mom's age and her dad's age equals to 70. She is wondering when the sum of their ages will be equal to 100. Can you help her?
2. Andy took a number, added 1 to it, then subtracted 2, then multiplied by 3, and then divided by 4. The result was 6. What number did Andy start with?
3. Five apples cost more than six bananas. What is more expensive: six apples or seven bananas?
4. Sarah multiplied all the numbers starting with 3 and ending with 23. What is the last digit of the result? (*Hint*: Please do not multiply all these numbers!)

Homework

Adam and Beth live in an apartment building. They want to start at the top floor, go down, and come back to the top floor. Adam is taking the elevator. Beth is taking the stairs.

- When Beth runs down the stairs, she is twice as fast as the elevator;
- When Beth runs up the stairs, she is twice as slow as the elevator.
- The speed of the elevator is the same independently of whether the elevator goes up or down.

Who will come back to the top floor first? Make pictures to explain your solution.