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Warm-up

Problem 1 *Many, many years ago in England, a man inherited some money and used it to start a business. In the duration of three consecutive years, he was spending £100 on the annual costs of running the business and gaining a third of what was left after taking away £100. By the end of the third year, the man doubled the original capital. What was the original capital?*

Problem 2 *Ten children are playing in a chess tournament. Each of them plays with all others. How many games do they play in total?*

Problem 3 *Which is greater, the sum of all even numbers from 0 to 10,000 or the sum of all odd numbers from 1 to 9,999? By how much? Please answer the questions without computing the sums.*

Problem 4 *Draw a piecewise-linear line that has the following properties.*

- 1. It is closed (the endpoint of the last segment is the beginning of the first).*
- 2. It has 6 segments.*
- 3. Each segment intersects only one of the remaining segments at a point different from the endpoints of the segments.*

Is a similar figure with 7 segments possible? If you think it is, please draw it. If you think it isn't, please explain why.

Problem 5 *Please draw a 4D triangular pyramid.*

Problem 6 *Please draw a 9D triangular pyramid. Hint: this problem is closely related to Problem 2.*

Time to get back to the algebra of logic...

Problem 7 *If possible, please expand and then simplify the expressions below according to the rules of the corresponding algebras.*

Algebra of Polynomials

$$1 + 1 =$$

$$a + (-a) =$$

$$w \times (-w) =$$

$$x(x + 1) =$$

$$(x + y)(x + y) =$$

$$(x + (-y))(x + (-y)) =$$

$$-\underbrace{(s + \dots + s)}_{56 \text{ times}} =$$

Boolean Algebra

$$1 + 1 =$$

$$a + (\neg a) =$$

$$w \times (\neg w) =$$

$$x(x + 1) =$$

$$(x + y)(x + y) =$$

$$(x + (\neg y))(x + (\neg y)) =$$

$$\neg \underbrace{(s + \dots + s)}_{56 \text{ times}} =$$

The problem continues on the next page.

Algebra of Polynomials

$$-\underbrace{(p \times \dots \times p)}_{65 \text{ times}} =$$

$$a \times (a + b) =$$

$$(x + y)(x + z) =$$

$$(x + y)(x + (-y)) =$$

$$(x + y)(x + y)(x + y) =$$

Boolean Algebra

$$\neg \underbrace{(p \times \dots \times p)}_{65 \text{ times}} =$$

$$a \times (a + b) =$$

$$(x + y)(x + z) =$$

$$(x + y)(x + (\neg y)) =$$

$$(x + y)(x + y)(x + y) =$$

Problem 8 Which algebra, Boolean or polynomial, does the following identity belong to? Why?

$$(x + y)(x^2 - xy + y^2) = x^3 + y^3$$

Prove the identity in the space below.

Problem 9 *On the Island of Knights and Liars, knights always say the truth, liars always lie. The following conversation between Alice, Bob, Charlie, and Daniel takes place on the island.*

Alice to Bob – You are a liar!

Bob to Charlie – You are a liar!

Charlie to Daniel – They are both liars and so are you!

Who are knights and who are liars?

Problem 10 *Four boys, Peter, Quentin, Robert, and Samuel, competed in a 100 m race. The next day, they made the following statements.*

Peter – I was neither first nor last.

Quentin – I was not the last.

Robert – I was the first.

Samuel – I was the last.

Three of them said the truth, one of them lied. Who lied and who won the race?

Recall that the DNF, Disjunctive Normal Form, of a Boolean algebra expression is an equivalent expression that is a sum of products of the elementary statements and their negations.

Problem 11 *Find the DNF of the following expression.*

$$\neg(\neg AB) + \neg((A + B)(A + C) + \neg B + \neg C) =$$

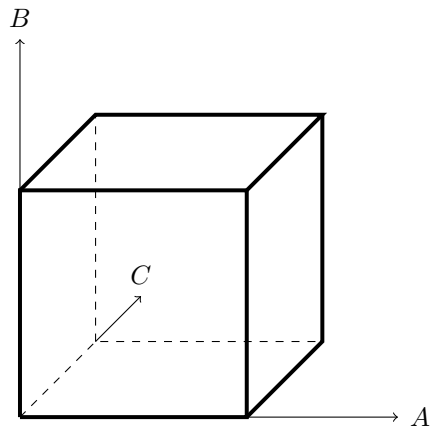
Recall that the FDNF, Full Disjunctive Normal Form, is the DNF such that each product has all the variables (or their negations).

Problem 12 *Find the FDNF of the Boolean algebra expression from the previous problem.*

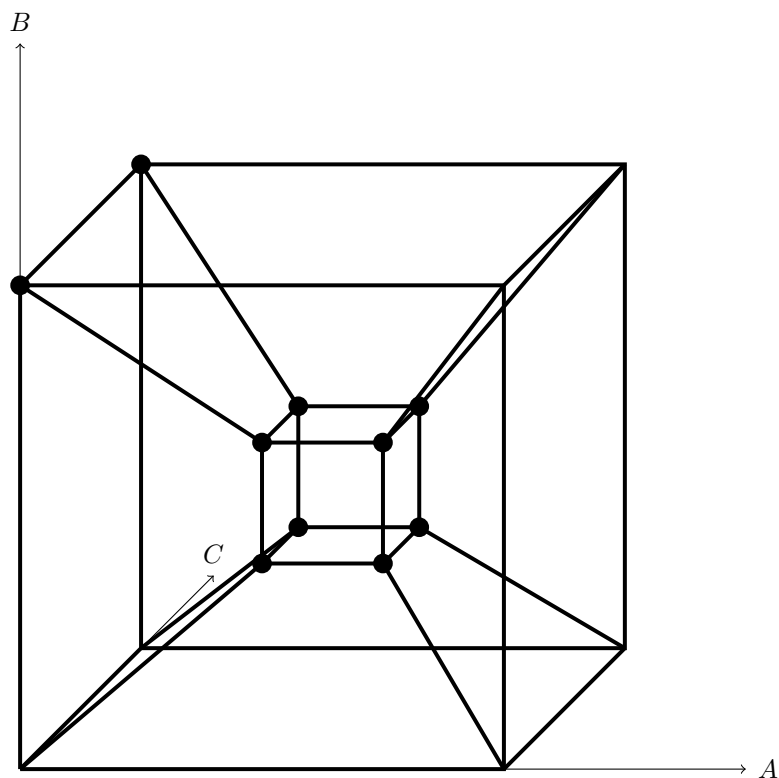
Problem 13 *Mark the vertices of the cube below by the products from the following FDNF expression.*

$$A\bar{B}C + A\bar{B}\bar{C} + \bar{A}BC + \bar{A}B\bar{C} + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C}$$

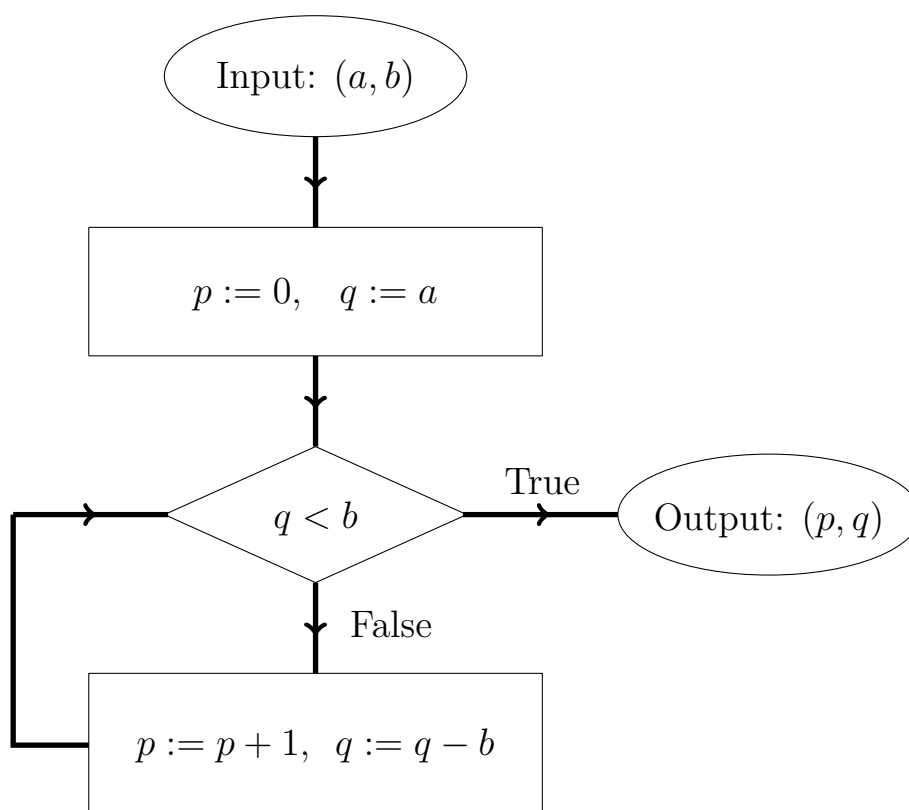
Use the geometric approach to simplify the sum – first simplify the sums that mark the edges, then the sums that mark the 2D faces of the 3d cube.



Problem 14 Write down the *FDNF* expression corresponding to the marked vertices of the following tesseract. Then use the geometric approach to simplify.



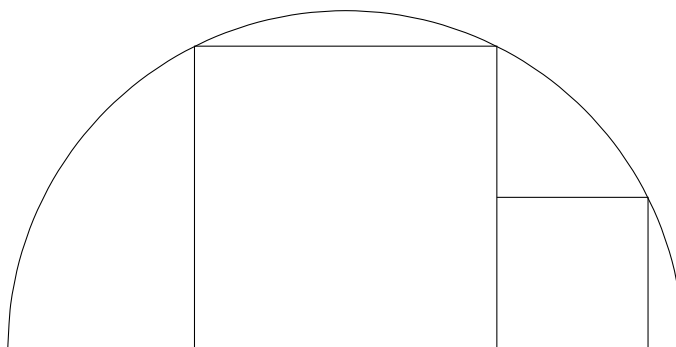
Recall that the algorithm below divides a positive integer a by a positive integer b and finds the quotient p and the remainder q .



Problem 15 Use the algorithm to divide 1111_2 by 100_2 without switching to the decimals.

If you are finished doing all the above, but there still remains some time...

Problem 16 *Prove that the area of the larger square below is four times greater than the area of the smaller square.*



Problem 17 *Someone slides a comb having six teeth per centimeter against a stationary comb that has five teeth per centimeter. While sliding, the experimentator looks at a source of light through the combs. For each comb, all the teeth have equal width and so do the gaps between them. At what speed would the lit spaces (the spaces where the light gets through the combs' teeth) be moving, if the sliding comb moves at the speed of 1 centimeter per second?*

