

Egyptian Multiplication

Beginners Circle 11/9/2014

Ancient Egyptians had an interesting method for multiplying two numbers. Suppose that you have to multiply two numbers (e.g., 23 and 18). The basic operation for them was multiplying a number by 2. (In other words, adding a number to itself). They reduced all other multiplication problems to it. Here is how they would start multiplying 23 by 18 (in modern notation):

23	18
—	—
1	18
2	36
4	72
8	144
16	288

Here is what they did to complete the multiplication

1. Below the first number (in this case, 23), they would write all of the powers of 2 that are smaller or equal to the first number.
2. In the second column, they would keep doubling the second number (in this case, 18).
3. After that, they would represent the first number as the sum of the powers of 2 (so that each of the powers of 2 is used at most once).
For example, if the first number is 23, they would find

$$23 = 16 + 4 + 2 + 1.$$

After that, they would mark those rows where these powers of 2 are present in the left column. (In our example, the first, the second, the third, and the fifth rows are marked).

Finally, all there is to do at this point is to add the marked numbers in the second column:

		1	8
		3	6
		7	2
+	2	8	8
	—	—	—
	4	1	4

Thus, the result of the multiplication is 414.

1. (a) Multiply 13 by 22 using Egyptian multiplication.

(b) Find the binary representation of 13 using your computation in part (a).

(c) Did you start with the highest or the lowest power of 2?

2. Explain how each number in the second column is obtained from the number in the first column, which is in the same row.

E.g. How do you get

- 72 from 4 and 18?

- 144 from 8 and 18?

3. Using what you noticed in question 2 do the following:

(a) Rewrite each term in the sum: $18 + 36 + 72 + 144$ as a product of 18 and a power of 2.

- $18 =$

- $36 =$

- $72 =$

- $144 =$

(b) Rewrite the whole sum:

- $18 + 36 + 72 + 144 =$

(c) What do you notice? Can you simplify this expression by factoring out 18?

4. Multiply the following numbers using Egyptian Multiplication:

(a) 13×41

(b) 41×13

5. Given two numbers, which one (smaller or larger) will you use as the first number in Egyptian Multiplication? Why? Give an example to justify your answer.

6. Explain in your own words how Egyptian Multiplication works.

7. With a partner, have a race to see who can multiply numbers faster. One of you must use Egyptian Multiplication and the other must use regular, long multiplication. Race 6 times alternating the type of multiplication you do. Show your work below:

(a) 25×31

(b) 38×45

(c) 12×63

(d) 17×52

(e) 112×85

(f) 256×50

8. Do you like Egyptian Multiplication or long multiplication better? Why?

1. Multiply 13×22 using Russian Peasant Multiplication.

2. Using what you notice on the previous page, 1 do the following:

(a) Rewrite each term in the sum: $18 + 36 + 72 + 144$ as a product of 18 and another number:

- $18 =$

- $36 =$

- $72 =$

- $288 =$

(b) Rewrite the whole sum using the expressions above:

- $18 + 36 + 72 + 288 =$

(c) What do you notice? Can you simplify this expression by factoring out 18?

Note that in this algorithm we are starting with the lowest power of 2 present in the number 23.

3. Let's learn to write numbers as sums of powers of 2 (without repetition) *starting with the smallest power of 2* by using Russian Multiplication. Recall how Russian Peasant Multiplication works for 23×18 .

Is this row used?	23	18	Multiplication Factor
			1
			2
			4
			8
			16

- (a) Write the values in the column for 23 by using the following results of division by 2:

$$23 \div 2 = 11 \text{ R } 1$$

$$11 \div 2 = 5 \text{ R } 1$$

$$5 \div 2 = 2 \text{ R } 1$$

$$2 \div 2 = 1 \text{ R } 0$$

$$1 \div 2 = 0 \text{ R } 1$$

- (b) Write the values in the column for 18 (each next value is the double of the previous value);
- (c) In the column labeled "Is this row used?" write 1 if the row is used and 0 if it is not used in Russian Peasant Multiplication. (Remember that a row is used in the Russian Peasant Multiplication if the number in the first column is odd)
- (d) What do the numbers in the column "Multiplication Factor" add up to (only using rows that are used)?

- (e) Write down 23 as the sum of powers of 2 starting with the smallest power:

- (f) Using your answers, write down the binary representation of 23.

4. We now have an algorithm of writing a number as a sum of powers of 2 starting with the smallest power.

—	27	—

- (a) At each step, divide the number in the middle column by 2 and write the result just below it (disregarding the remainder).
- (b) If the number you got in the previous step is even, write 0 in the left column. If the number is odd, write 1 in the left column.
- (c) In the right column, write the powers of 2, starting with $2^0 = 1$.
- (d) Circle the powers of 2 that are in the rows where the left column has 1.
- (e) The circled numbers in the right column add up to the number you started with.
- (f) The numbers in the left column (starting with the top number) form the binary representation of the number you started with.
5. To understand why this algorithm works, answer the following questions:
- (a) What is the rightmost digit (0 or 1) in the binary representation of an even number?
- (b) What is the rightmost digit (0 or 1) in the binary representation of an odd number?
6. Suppose you start with a binary number written with n digits. Using arithmetic operations, how do you get the number written by the first $(n - 1)$ digits? (*Hint*: you might want to consider the cases of even and odd numbers separately). Explain the connection between the ideas you discovered in problems 5 and 6 and the algorithm of converting a number into binary notation using division by 2 that you saw in problem 4.

7. Write the following numbers as sums of powers of 2 (binary notation) starting with the smallest power (Hint: Use the method of problem 4):

(a) $9 =$

$$9 \div 2 = R$$

(b) $14 =$

$$14 \div 2 = R$$

(c) $23 =$

$$23 \div 2 = R$$

(d) $44 =$

$$44 \div 2 = R$$

8. Use the Russian Peasant multiplication in the following examples:

(a) $27 \times 23 =$

Is this row used?	27	23	Multiplication Factor

(b) $33 \times 22 =$

	33	22	

(c) $19 \times 45 =$

	19	45	

RACE!

9. Represent each number below as a sum of powers of 2. One person uses the Egyptian method (which starts with the largest power of 2 first and multiplication by 2). The other person uses the Russian Peasant method (which starts with the smallest power of 2 first and division by 2).

(a) 26

(b) 45

(c) 52

(d) 68

(e) Which method do you like better, and why?

10. Now we know three methods of multiplication: Egyptian Multiplication, Russian Peasant Multiplication, and the “normal” long multiplication.

(a) Give an example where Egyptian Multiplication may be easier than the other two methods. Explain why.

(b) Give an example where Russian Peasant Multiplication may be the easiest method, and explain why.

(c) Give an example where the regular long multiplication may be the easiest method, and explain why.

11. Compare Russian Peasant Multiplication, Egyptian Multiplication and regular, long multiplication. What are the key differences? Which requires you to remember the most facts? Use the columns below to write the advantages for each type of multiplication.

Russian Peasant Multiplication	Regular, Long Multiplication	Egyptian Multiplication