

Sequences and averages!

November 2, 2014

1. Write or draw an example of a *sequence*. Be sure to note the pattern of rules it follows.
2. You are probably familiar with the following sequence of numbers:

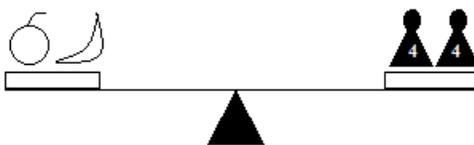
1, 2, 3, 4, 5, 6, 7, ...

Think about how the numbers above are related to each other. What is the “rule” of the pattern for the sequence?

- (a) The three dots (...) tell us that the pattern continues on. Write the next three numbers in the sequence.
- (b) Pick a number in the sequence above (e.g. 5). Relate it to the two numbers just before and just after it. You may find the number line below useful.



- (c) It is also true that any number in the sequence above is the *average* of the two numbers around it. What does this mean? Consider two fruits, an apple and a banana. The apple weighs 3 ounces, the banana weighs 5 ounces. What is their *average* weight? They are placed on a scale with two equal weights on the other side. How much does each weight have to be so that the scale is balanced?



Therefore, we can write

$$\text{apple} + \text{banana} = 2 \times \text{weight}$$

We can also write this as

$$\frac{\text{apple} + \text{banana}}{2} = \text{weight}$$

When we can replace the two objects with two equal identical objects (of equal weight), the weight of one of the identical objects is the *average* of the two.

- i. What is the average weight of the apple and banana above (where the apple weighs 3 ounces and the banana weighs 5 ounces)?
- ii. Do you think that two other fruits with different weights can still have the same *average* weight? Why or why not?

- iii. If the apple weighs 2 ounces and the banana weighs 6 ounces, what is the *average* weight of the fruits?
- iv. What is the *average* weight of an apple which weighs 4 ounces and a banana which weighs 10 ounces?
- (d) Pick a number in the sequence 1, 2, 3, 4, 5, 6, ...
Show it is the *average* of the two numbers surrounding it.
- (e) Can you find another triple of numbers in this sequences so that the second is the average of the first and third? How do these numbers look in picture form, like above?

3. Consider the following sequence of numbers:

$$2, 4, 6, 8, 10, \dots$$

- (a) How do you go from one number to the next number? Write out the next few numbers of the sequence.
- (b) Pick a number in the sequence 2, 4, 6, 8, 10, ... and show it is the *average* of the numbers which surround it.

4. Consider the following sequence:

$$29, 25, 21, 17, \dots$$

- (a) How is this sequence of numbers different from the previous two?
How do you go from one number to the next number?

- (b) Write the next 3 numbers in the sequence.

- (c) We found in #2 that any number in that sequence
 $1, 2, 3, 4, 5, 6, \dots$ was the *average* of the two numbers
around it. Is this true about the numbers in this sequence? Check
with the numbers you added to the sequence in part (b) above.

5. Find the pattern in the following sequence and write out the next 3 numbers:

$$15, 10, 15, 10, 15, 10, _, _, _, \dots$$

- (a) Are the numbers of this sequence *averages* of the two surrounding numbers as they were in some of the previous questions?

6. Below are some more interesting patterned sequences of numbers. Find the rule for the pattern and write the next few numbers of the sequence.

(a) 1, 0, 1, 1, 0, 1, 1, 1, 0, ...

i. Next 7 numbers:

(b) 1, 2, 4, 8, ...

i. Rule for pattern:

ii. Next 5 numbers:

(c) 1, 3, 6, 10, 15, ...

i. Rule for pattern:

ii. Next 5 numbers:

iii. Where are the odd numbers and even numbers located in this sequence? How do you know?

(d) 1, 4, 9, 16, 25, ...

i. Rule for pattern:

ii. Next 3 numbers:

iii. Can you explain why every other element is an odd number?

(e) 10, 11, 9, 12, 8, 13, 7, 14, ...

i. Rule for pattern:

ii. Next 5 numbers:

(f) In any of the above sequences, are the numbers *always* the average of their two surrounding numbers? The sequences were:

1, 0, 1, 1, 0, 1, 1, 1, 0, ...

1, 2, 4, 8, ...

1, 3, 6, 10, 15, ...

1, 4, 9, 16, 25, ...

10, 11, 9, 12, 8, 13, 7, 14, ...

(g) Using your answers to questions about averages above, can you make a guess as to *when* numbers in sequences will be the averages of their two surrounding numbers?

Challenge

1. Fill in the blank spots in the sequence:

5, 6, __, __, __, 20, 26...

2. The following two sequences are related. See if you can figure out how, and write out the next few numbers of the sequences:

1, 8, 17, 30, 49...

7, 9, 13, 19, 27...

3. Jill and Jim are twins. At the age of 2, their average weight is 12 kilograms. In the next year, Jill gains 2 kilograms and Jim gains 4 kilograms. What is their average age when they are 3 years old?
4. The average weight of Mark and Mina is 25 kilograms. A year later, their average weight is 30 kilograms. During this year, Mina gained 2 kilograms. How many kilograms did Mark gain?
5. John takes two exams, and his average score is 84. He then gets a score of 100 on the third exam. What is his new average score?
6. After taking two exams, Jane's average score is 88. What score must Jane make on the third exam so that her new average score is 90?
7. Sarah and Sam have a pet dog who always stands exactly between the two of them. Sarah moves 5 meters to the right and Sam moves 9 meters in the same direction. How far does the dog move?
8. Sarah then moves 4 meters to the left, while Sam moves 10 meters to the right. Which direction does the dog move? How far does the dog move this time?