

Oleg Gleizer
prof1140g@math.ucla.edu

Problem 1 Is the number N_{2012} of rectangles having integral side lengths and the perimeter 2012, greater, less, or equals to the number N_{2014} of rectangles having integral side lengths and the perimeter 2014? Please put the correct sign in the box below and explain the reason.

$$N_{2012} \boxed{} N_{2014}$$

Problem 2 *Prove that among any six integers there exist two such that their difference is divisible by five.*

Problem 3 *Prove that among any $n+1$ integers there exist two such that their difference is divisible by n .*

Problem 4 *An old clock gets one minute behind time every hour. It is showing the correct time at the moment. When will it show the correct time next time?*

Problem 5 *The product of three numbers equals 1000. The first two factors were increased 10%, the the third was decreased 20%. How did the product change?*

Problem 6 *A shorter man's steps are 20% shorter than a taller man's steps, but the shorter man makes 20% more steps as he walks. Which of the men walks faster?*

Problem 7 *Alice, Bob, and Charlie ran a 100 meters race. When Alice finished, Bob was 10 meters behind her. When Bob finished, Charlie was 10 meters behind him. Assuming that they all ran at a constant speed, where was Charlie when Alice finished the race?*

Problem 8 *Two stores had equal prices. The first store decreased all the prices 10% one day and 10% the next day. The second store decreased all the prices 20% right away. What store (out of these two) would you go to?*

Problem 9 *The Pacific Ocean water has 3.5% of salt (with respect to weight). How much distilled water does one need to add to the Pasific Ocean water to reduce the amount of salt in the solution to 0.5%?*

Problem 10 Find the smallest natural number n such that there exists a fraction m/n in between the numbers 0.4 and 0.5.

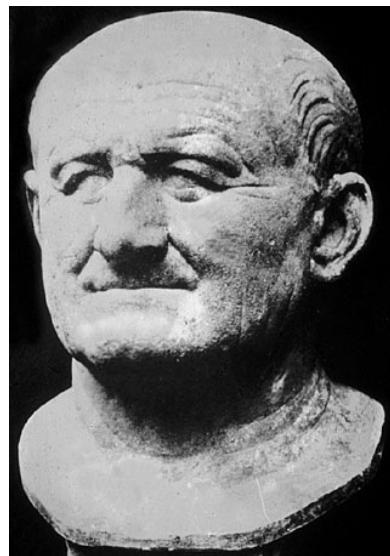
The following topic was suggested by one of our instructors, Eric Auld.

A Jewish general turned Roman historian, Josephus Flavius, lived in the first century AD. His book *The Judean War* describes a Jewish rebellion against the Romans that ended up with the destruction of Jerusalem in AD 70.

During the war, Josephus and his forty soldiers were trapped in a cave, with the Romans blocking the exit. The Jews chose suicide over capture. They decided to form a circle. Then a fighter would kill a neighboring fighter on his left with a blow of his sword and pass the sword further left until there was one man left standing. That one was supposed to kill himself. The last one happened to be Josephus who, instead of committing a suicide, turned himself over to the Romans.¹

¹We have slightly modified the story from the book to make it closer related to the subject of our interest, binary numbers.

Josephus proceeded to become a friend of the Roman emperor Vespasian Flavius (hence Josephus's Latin last name) and a popular historian.



Vespasian Flavius

The above story gave rise to a family of combinatorial problems all bearing the common name, the *Josephus Problem*.

Problem 11 *What was the position of Josephus in the circle?*

Problem 12 Suppose that there are n soldiers, including Josephus, in the cave. What should the position of Josephus be in order for him to stay alive? Hint: use binary numbers instead of the decimals.

If you are finished doing all the above, but there still remains some time ...

When you watch a moving car on the screen of a cinema theatre, sometimes the wheels of the vehicle seem to rotate the right way, but sometimes they seem to rotate the opposite way. Understanding this phenomenon is a part of solving the following problem.

Problem 13 *A film runs through the projector at the rate of sixteen frames per second. On the screen, you see a moving car. In real life, the diameter of the car's tires is one meter. In the movie, the car's wheels rotate four times per second. What is the real life speed of the car?*