

## LAMC Intermediate I & II

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### A solution to the chicken problem

**Problem 1** *Three farmers went to the market to sell chicken. One farmer brought 10 chicken to sell, the second brought 16, while the third brought 26. The farmers agreed to charge the same price for their animals. After the lunch break, afraid that they may not sell all of the chicken, the farmers lowered the price. As a result, they sold all the chicken, making \$35 each. What was the price before and after the lunch break?*

**Solution** Let  $p$  be the price before the lunch and let  $q$  be the price after the lunch.

$$p > q > 0 \tag{1}$$

Let  $x$ ,  $y$ , and  $z$  be the number of chicken the first, second, and third farmer have sold before the lunch respectively.

$$0 \leq x \leq 10 \quad 0 \leq y \leq 16 \quad 0 \leq z \leq 26 \tag{2}$$

The number of chicken the first, second, and third farmer have sold after the lunch is  $10 - x$ ,  $16 - y$ , and  $26 - z$  respectively. The fact that each of the farmers has earned \$35 is described mathematically by the following three equations.

$$px + q(10 - x) = 35 \tag{3}$$

$$py + q(16 - y) = 35 \quad (4)$$

$$pz + q(26 - z) = 35 \quad (5)$$

Opening up parenthesis in 3 gives us

$$px + 10q - qx = 35$$

which we rewrite as follows.

$$(p - q)x + 10q = 35 \quad (6)$$

Similarly, 4 and 5 give us the equations below.

$$(p - q)y + 16q = 35 \quad (7)$$

$$(p - q)z + 26q = 35 \quad (8)$$

Subtracting 6 from 7 gives us the following.

$$(p - q)(y - x) + 6q = 0 \quad (9)$$

Since  $p > q > 0$ , this is only possible if  $y - x < 0$ , or  $x > y$ . Subtracting 7 from 8 gives us the following.

$$(p - q)(z - y) + 10q = 0 \quad (10)$$

Once again,  $p > q > 0$ , so  $z - y < 0$ , or  $y > z$ . We arrive at the following inequalities bounding  $x$ ,  $y$ , and  $z$ .

$$x > y > z \geq 0 \quad (11)$$

Let us consider the difference between 8 and 6

$$(p - q)(z - x) + 16q = 0 \quad (12)$$

next to 10.

$$(p - q)(z - y) + 10q = 0$$

Let us rewrite this pair of equations as

$$\begin{aligned} 16q &= (p - q)(x - z) \\ 10q &= (p - q)(y - z) \end{aligned}$$

and let us divide the first equation by the second.

$$\frac{16}{10} = \frac{x - z}{y - z}$$

We can rewrite the latter formula as follows.

$$8(y - z) = 5(x - z) \tag{13}$$

According to 13, the number  $x - z$  must be a multiple of 8. Since  $10 \geq x > z \geq 0$ , we have  $10 \geq x - z > 0$ . Therefore,  $x - z = 8$  or  $x = z + 8$ . Since  $10 \geq x$  and  $z \geq 0$ , the variable  $z$  can only take three values, 0, 1, and 2.

Let us take another look at 13. Since  $x - z = 8$ , we have  $y - z = 5$ , or  $y = z + 5$ . We arrive at the following table.

$x = z+8$	$y = z+5$	$z$
8	5	0
9	6	1
10	7	2

Let us consider the table case-by-case.

Case 1:  $x = 8, y = 5, z = 0$ . Plugging these numbers into 3, 4, and 5 gives us the following equations.

$$\begin{aligned} 8p + 2q &= 35 \\ 5p + 11q &= 35 \\ 26q &= 35 \end{aligned}$$

The last equation means that

$$q = \frac{35}{26} = 1.3461\dots$$

The price in cents must be an integer, so the first case does not work.

Case 3:  $x = 10$ ,  $y = 7$ ,  $z = 2$ . Plugging these numbers into 3, 4, and 5 gives us the following equations.

$$\begin{aligned}10p &= 35 \\7p + 9q &= 35 \\2p + 24q &= 35\end{aligned}$$

The first equation means that  $p = 3.5$ . Then according to the second equation,

$$q = \frac{35 - 7 \times 3.5}{9} = 1.1666\dots$$

The third case does not work either.

Case 2:  $x = 9$ ,  $y = 6$ ,  $z = 1$ . Plugging these numbers into 3, 4, and 5 gives us the following equations.

$$\begin{aligned}9p + q &= 35 \\6p + 10q &= 35 \\p + 25q &= 35\end{aligned}$$

The first equation gives us  $q = 35 - 9p$ . According to the second equation,  $6p + 10(35 - 9p) = 35$ , or  $84p = 315$ . So,  $p = \$3.75$ . Thus  $q = 35 - 9 \times 3.75$ , or  $q = \$1.25$ . To double-check, let us plug both numbers into the third equation.  $3.75 + 25 \times 1.25 = 35$  indeed.

Finally, the problem is solved. The price before lunch was \$3.75, the price after lunch was \$1.25.