How To Park Your Car

David Patrick
patrick@artofproblemsolving.com

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Today we'll work on a few problems that have a common theme: parking cars. (For some reason, this theme seemed appropriate to me while I am visiting Los Angeles.)

Despite the common theme, the math in the various problems is somewhat different, and the problems are mostly unrelated.

Questions. . .

While you're thinking about these problems, consider the following questions:

- How can you get started on these problems?
- What tactics can you try to explore these problems?
- Can you alter the problems in any way to make them more approachable?
- How do these problems generalize? For instance, what if I replaced the given numbers with much larger numbers in the various problems: how would this change how you solve them? What's the long-range behavior?

These are good questions to ask when trying to solve *any* problem, but you might find them especially useful for the problems that follow.

Problem #1: The Lazy Valet Attendant

Suppose 8 people go out to dinner. Each drives separately and gives his or her car keys to the valet parking attendant.

At the end of the meal, the 8 people go to retrieve their cars. The valet attendant, being lazy, didn't bother to properly tag the keys when the cars were dropped off, so he just randomly gives a set of keys to each of the 8 people.

What's the probability that none of the 8 people receives the correct set of keys?

Problem #2: Monster Truck Rally

A parking lot has 16 spaces in a row. 12 cars arrive, each of which requires one parking space, and their drivers choose spaces at random from among the available spaces. Shannon then arrives in her monster truck, which is so wide that it requires 2 adjacent spaces to be able to park.

What is the probability that Shannon is able to park? (And no, crushing some cars to make room is not considered "able to park.")

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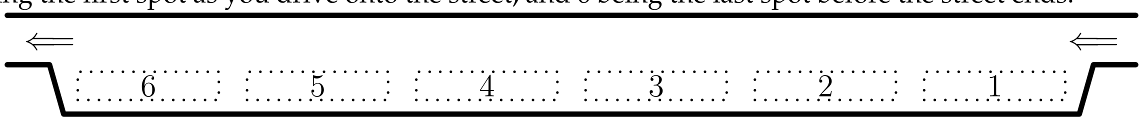
Problem #3: The Poorly-Designed Parking Lot

Suppose 6 people live on a narrow one-way street that has six street-parking spots labeled 1 through 6, with 1 being the first spot as you drive onto the street, and 6 being the last spot before the street ends.



Each person has his or her favorite parking spot, and will always first check if his or her spot is free; if not, he or she will take the first available spot after his or her favorite (backing up is not an option!).

Assume each person chooses his or her favorite parking spot independently and at random.

The street starts empty and they return home one-at-a-time.

(a) What's the probability that each person has a different favorite parking spot?

(b) What's the probability that they all park successfully?

Problem #4: Parallel Parking Wars

For the sake of this problem, assume that every car is exactly 5 meters long, and that everyone is an expert parallel parker: if there is a 5-meter-long gap in the curb, then anyone can park their car there. (Those of you who have ever tried parallel parking know that this is not the case in the real world.)

Outside the AoPS office in San Diego is a 50-meter-long stretch of curb. Suppose that the curb starts empty, and that when cars arrive that wish to park outside our office, they randomly pick a position on the curb into which they'll fit, without regard for what other cars might already be parked.

What is the expected number of cars that will be able to park before a car arrives that is unable to park, because there is insufficient room?

Note: I don't expect you to solve this. It's a very hard problem and you probably need some advanced calculus to get an exact answer; it's not even clear that it's possible to get an exact answer! But try to develop an intuition about the problem, and see if you can get some partial results. For example, what if we had a 10-meter-long curb? How about a 15-meter-long curb? You should be able to solve those, but it starts to get really hard with a 20-meter-long curb.

Acknowledgements: Problem #2 is from the 2008 AMC 12B. (I changed the name and the vehicle.) Problem #3 is based in part on the notes *Where Can I Park?* by Elgin Johnston and *Parking Functions* by Matthias Beck, and the session *To Park or Not To Park?* presented by Pari Ford at the 2012 Julia Robinson Math Festival in Washington, DC. Problem #4 was studied by the Hungarian mathematician Alfréd Rényi.