

INTRODUCTION TO PROOFS

MATH CIRCLE (HS) 10/12/2014

Example Problem 1

Suppose you have two boxes. Box A contains 100 white balls and box B contains 100 black balls. You take half of the balls (randomly) from box A and put them into box B . Then take 50 balls (randomly) from box B and put them into box A .

What can you say about the number of white balls in box A compared to the number of black balls in box B ?

Answer

The number of white balls in box A is *always equal* to the number of black balls in box B .

But can we prove it?

1) Critique the following “proofs”. Should any be considered a proof?

a) The two numbers are the same, because however many black balls we move to box A , that’s how many white balls have to stay in box B .

b) If box A ends up with fewer than 50 black balls, then some of the balls that we moved to it must have been white. So box B will have fewer than 50 white balls left, since some of them got returned to the white box. The numbers must match up, since we moved 50 balls each time, so the final number of white balls in box A is the same as the black balls in box B .

c) When we first move the 50 white balls to box B , the fraction of white balls in the black box is $\frac{1}{3}$ (50 out of 150). When we move 50 balls back to box A , roughly 17 of them will be white (and 33 black). Thus, box A ends up with $50 + 17 = 67$ white balls and box B ends up with $100 - 33 = 67$ black balls as needed.

2) In small groups come up with your own proof to Example Problem 1. Be prepared to present it to the class.

Example Problem 2

How many prime numbers are there?

Answer

The famous result of Euclid states there are infinitely many primes.

3) Prove that there are infinitely many primes. Be prepared to present it to the class. (Make note of anything you are taking for granted in your proof.)

4) Compare/contrast the methods of proof used for Example Problems 1 and 2.