RELAY SOLUTIONS!

LAMC INTERMEDIATE - 3/16/14

(1) Write down a number between 1 and 1000. The team with the second largest and the team with the second smallest number each receive 2 points. **Answer:** Varies

(2) A man is looking at a portrait. Referring to the person in the portrait, the man says, "Brothers and sisters I have none, but this man's father is my father's son." Who is the person depicted in the portrait? **Answer:** His son

(3) Seven friendly hobbits are neighbors in the Shire. Initially, two hobbits have blue houses and five hobbits have red houses. Each day, a different hobbit exits his house and looks around. If his house is red, and more of his friends have blue houses than red houses, he will repaint his house blue. Similarly, if his house is blue, and more of his friends have red houses than blue houses, he will repaint his house red. What is the eventual state of the seven houses? On average, when will this state be accomplished? (1 point, 2 points) **Answer:** all red. In 5.3 days (accept $\frac{6(7)+5(6)+4(5)+3(4)+2(3)+2}{21}$ days, if they say something like 5 or 6 try to find out if they are rounding this number).

(4) At some point in the past, McDonalds sold McNuggets in packs of 6, 9, and 20. What is the largest number of McNuggets you could not buy? Or is there any such number? **Answer:** 43.

(5) What is the Queen dominating number on a 7×7 board? On an 11×11 board? (2 points, 2 points) **Answer:** 4, 5.

(6) Evaluate $\sum_{n=1}^{\infty}(\frac{5}{7})^n$. **Answer:** $\frac{5}{2}$

(7) **(Game)** Choose one competitor from your team to play 24. In this game, 4 cards are placed on the table and the competitors must use addition, subtraction, division, and multiplication to combine their values to reach the number 24. Each team's representative will play against each other team's representative. (Win 1 point, Lose -1 point, Draw 0)

(8) Give a "zero-knowledge" solution to Sudoku. That is, come up with a way prove (with zero doubt) to your instructor that you have solved a Sudoku puzzle without providing your instructor any help to solve the puzzle himself. **Answer:** Varies, I can provide one if you want.

(9) Suppose you flip 27 coins. What is the probably that an even number of them are heads? (1 point for answer, 1 point for a sensible explanation) **Answer:** $\frac{1}{2}$. Example Explanation: having an even number of heads is the same as an odd number of tails (because 27 is ODD!), which is of course the same as an odd number of heads by symmetry.

(10) A town has two circular ponds. The first pond has radius 2 and center $(0, 2)$ while the second pond has radius 5 and center at $(15, 10)$. What is the shortest distance between the edges of the two ponds? **Answer:** 10

(11) Give a sentence in everyday language (which does not include the symbol x) which has the same meaning as the following. Let $P =$ "x is a dog," $Q =$ "x has a good sense of smell," and $R =$ "x hates cats."

$$P \to (Q \wedge R)$$

Answer: All dogs have a good sense of smell and hate cats. (other sentences also possible)

(12) Which triangle has the smallest perimeter for a given area? (Note: This is the "opposite question" to what we studied in class... the triangle with the largest area for a given perimeter.) **Answer:** Equilateral of course.

(13) Notice that $4! \geq 2^4$ and $5! \geq 2^5$. Does this pattern continue for $n > 5$? If so, prove it. If not, provide a counter-example. **Answer:** Yes. Any mention of the word induction is sufficient for points on the proof.

(14) **(Game)** Send at least two members of your team to play a game of mathematical pictionary. Points awarded for each correct guess by the non-drawing players.

(15) One point if you can spell every instructor's first name. A bonus point for each instructor whose name you can spell in full (first and last). **Answer:**

(16) The maximum number of regions the plane can be divided into by two straight lines if 4. What is the maximum number of regions the plane can be divided into by 4 straight lines? By 5 straight lines? (2 points each) **Answer:** 11, 16.

(17) What is the maximum number of regions the plane can be divided into by n straight lines? **Answer:** $\frac{n^2+n+2}{2}$.

(18) Convert the following fraction to a decimal: $\frac{14}{17}$.**Answer:** $.8235294117647058$ repeating

(19) Prove that the number of people through all of history who have shaken hands (with other people) an odd number of times is even. **Answer:** every time two people shake hands the total changes by +2, 0, or -2

(20) As you leave your house in the morning, you realize that you may have forgotten your phone. You think that 80% of the time you have your phone and 20% of the time you left it on the bathroom counter. Further, you decide that it is equally likely that you have the phone in your right pocket and your left pocket. You reach into your right pocket and find out the phone is not there. What is now the probability that the phone is in your left pocket? **Answer:** $\frac{2}{3}$ or 66%.

(21) There are three lottery tickets, one is a winner and two are losers. You pick randomly between the three tickets then your friend picks up another one and reveals that it is a loser. Your friend then offers to trade you tickets (he will give you the other one that has not been checked for the one you picked). Should you do it? Why or why not? Or does it matter at all? **Answer:** Yes. You trade and you have a $\frac{2}{3}$ chance of winning.

(22) You have three boxes. One box is labeled "Apples," one box is labeled "Oranges," and the third box is labeled "Apples and Oranges." All boxes are unfortunately labeled incorrectly, how can you fix the problem by removing only one piece of fruit from one of the boxes? **Answer:** Take one from the apples and oranges box, because you know it contains only apples or only oranges.

(23) In how many ways can an $n \times 2$ rectangle be covered by 1×2 dominoes? **Answer:** F_{n+1} (The $n+1$st fibonacci number)

(24) Write a Limerick with some mathematical content. (A Limerick will have 5 lines and have rhyme scheme AABBA.) Points awarded at the discretion of your judge. **Answer:** Varies

(25) How many terms of the sequence of natural numbers 1,2,3,4,... must be added to give a sum exceeding one million? **Answer:** 1414

(26) The Ackermann function is a rich source of counterexamples in logic because it grows too quickly. It is defined as follows

$$\begin{aligned} A(0,n) &= n+1 && \text{for every } n \geq 0 \\ A(m,0) &= A(m-1,1) && \text{if } m \geq 1 \\ A(m,n) &= A(m-1, A(m,n-1)) && \text{if } m,n \geq 1 \end{aligned}$$

Compute $A(5,0)$. (5 points) **Answer:** 65533.

(27) **Bonus:** The Ackermann function grows much more quickly when both m and n are not zero or one, for exampleGuess how many digits are in the number $A(4,3)$ when written out as a whole number. (5 points for the closest group, 4 points for the second closest, etc.) **Answer:** there are approximately 6×10^{19727} digits!

(28) Suppose that $ab = 3$ and $\frac{1}{a^2} + \frac{1}{b^2} = 4$. Compute $(a-b)^2$. **Answer:** 30

(29) If p, q, r are prime numbers such that their product is 19 times their sum, what is $p^2 + q^2 + r^2$? **Answer:** primes are 3,11, 19. So the desired value is 491.

(30) The pages of a book are numbered 1 through n. When the page numbers of the book are added, one of the page numbers was mistakenly added twice, resulting in the incorrect sum of 1986. What was the number of the page that was added twice? **Answer:** 33

(31) Prove that if x and y are positive, then $\frac{1}{x} + \frac{1}{y} \geq \frac{4}{x+y}$. **Answer:** Just check if the proof seems valid.

(32) Below are five statements. At most how many of them can be true together?
- (a) If b is true, then this statement is false.
- (b) If the number of true statements is greater than 2, then one of the true statements is c.
- (c) At least one of a and d is false.
- (d) Either b and c are both or they are both false.
- (e) b is true or false. ?**Answer:** 3, witnessed by a,d,e

(33) There is a unique polynomial $P(x)$ of the form $P(x) = 7x^7 + c_1x^6 + c_2x^5 + \cdots + c_6x + c_7$ such that $P(k) = k$ for $k = 1, 2, \ldots, 7$. Find $P(0)$.**Answer:** -35280.

(34) In any group of people, prove that there are two persons having the same number of acquaintances within the group. **Answer:** pigeonhole principle