

Week 7 Problems

LA Math Circle | Advanced Group

May 11, 2014

1. In the cube pictured below, determine the measure of angle AFC.

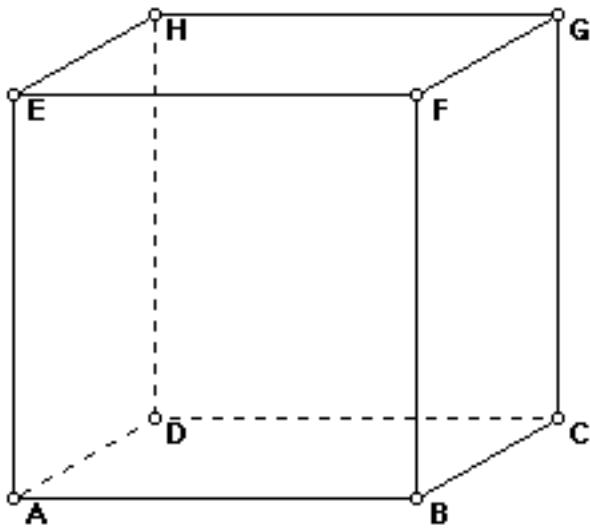


Figure 0.1: Problem 1.

2. Write the number 23 in base -2 (negabinary) notation. Then write the number 146 in base -3 notation. Generalize your procedure into a set of instructions somebody could follow to write an arbitrary number in terms of an arbitrary negative base. If you need to specify any restrictions, say so.

3. A school has 1,000 students and a row of 1,000 lockers. The lockers all start out closed. The first student walks down the line and opens every locker. The second student closes the even numbered lockers. The third student approaches every third locker and changes its state. If it was open he closes it; if it was closed he opens it. The fourth student does the same to every fourth locker, and so on through 1,000 students. (To illustrate, the tenth locker is opened by the first student, closed by the second, reopened by the fifth, and then closed by the tenth. All the other students pass by the tenth locker, so it winds up being closed.) **Once all students have passed through, how many lockers are open?**

4. A new two-mile long bridge has been proposed for construction over the San Francisco Bay, right along the waterline. The mayor vetoed the blueprint because it didn't allow for those big ol' 100-foot tall ships to pass underneath to the port.

The chief engineer replied, "Oh, no problem. I can re-design the bridge to let 200-foot ships pass underneath. The cars driving over won't even notice the difference – they'll only drive a few more feet than they would if the bridge were flat."

"Dude, what're you talking about?" asked the mayor. "No matter what you do, the bridge's span will be 400 feet longer – you gotta add in 200 feet up and 200 feet back down."

Well... who's right? If the new bridge isn't 400 feet longer, about how much longer is it? Make any appropriate assumptions and simplifications.