

SYMBOLIC LOGIC

(PART ONE)

LAMC INTERMEDIATE - 5/18/14

“Algebra” works by letting variables represent numbers and using connective symbols between variables like $+$, \times , $-$, and $=$. Symbolic logic works in exactly the same way. We will let variables represent “**propositions**” and use the connective symbols \neg , \wedge , \vee , \rightarrow , and \leftrightarrow .

WARM UP

Write down the meaning of each of the following:

• **Proposition:**

• **Propositional Variable:**

• \neg :

• \wedge :

• \vee :

• \rightarrow :

• \leftrightarrow :

• **Formula:**

FORMING MORE COMPLEX PROPOSITIONS

(1) Which of the following are propositions? Explain.

(a) The king of France enjoys ice skating.

(b) Dustin enjoys ice skating.

(c) All cats are black.

(d) The square root of 2 is irrational.

(e) Is this question hard?

(f) This sentence is false.

(2) If P and Q are propositional variables, which of the following formulas are propositions? Explain.

(a) $P \rightarrow Q$

(b) $\rightarrow P \wedge Q$

(c) $P \vee Q \leftrightarrow P \wedge Q$

(d) $Q \wedge \rightarrow \vee P$

(3) Let $P =$ “it is raining”, $Q =$ “Derek has an umbrella”, $R =$ “Morgan is wet”, $S =$ “Derek is wet”, and $T =$ “Miguel is drinking hot chocolate”. Translate each of the following statements into formulas.

(a) Derek has an umbrella.

(b) Derek does not have an umbrella.

(c) If it is raining, then Morgan is wet.

(d) Miguel is drinking hot chocolate if and only if it is raining.

(e) If it is raining and Derek does not have an umbrella, then Derek is wet.

(f) If Miguel is drinking hot chocolate, then Morgan is wet.

(g) If Derek has an umbrella or it is not raining, then Derek is not wet.

(4) Let $P, Q, R, S,$ and T be as in the previous problem. Translate the following formulas into English.

(a) Q

(b) $P \vee S$

(c) $P \vee S \rightarrow T$

(d) $P \wedge T \rightarrow Q \vee R$

(e) $\neg(S \vee T)$

(f) $\neg(S \wedge T)$

DETERMINING THE TRUTH VALUE OF PROPOSITIONS

If we know the truth value of propositional variables A and B , then we can determine the truth value of propositions which are built from them using logical connectives. For example, if we know that A is True and B is False, then we know that $A \wedge B$ is false.

- (1) Fill in the following truth table:

A	B	$A \wedge B$	$A \vee B$	$A \rightarrow B$	$A \leftrightarrow B$	$\neg A$
T	T					
T	F					
F	T					
F	F					

- (2) The truth table above was built from two propositional variables and contains 4 rows. How many rows will a truth table built from n propositional variables have?

- (3) Suppose that A is True, B is False, and C is True. Determine the truth value of the following:

(a) $(A \wedge B) \rightarrow C$

(b) $(\neg A \vee B) \wedge (C \rightarrow B)$

(c) $[(\neg A \vee B) \wedge (C \rightarrow B)] \vee [(B \rightarrow A \rightarrow C)] \leftrightarrow ((B \wedge A) \rightarrow C)$

(4) Two propositions are called **logically equivalent** if their truth value is always the same.

(a) Fill in the following truth table to show that A and $\neg(\neg A)$ are logically equivalent.

A	$\neg(\neg A)$
T	
F	

(b) Complete the following truth table to determine if $A \rightarrow B$ and $(\neg A) \vee B$ are logically equivalent.

A	B	$A \rightarrow B$	$(\neg A) \vee B$
T	T		
T	F		
F	T		
F	F		

(c) Construct your own truth table to show that $A \rightarrow B$ and $\neg B \rightarrow \neg A$ are logically equivalent. (Note: the second statement is called the **contrapositive** of the first)

- (5) This problem demonstrates why English teachers stress correct comma usage. Consider the following sentence:

“If it is raining then Deven is cold and Dustin is wearing a jacket.”

let R = “It is raining”, C = “Deven is cold” and J = “Dustin is wearing a jacket.”

(a) Insert a comma into the sentence so that it translates to $R \rightarrow (C \wedge J)$.

(b) Insert a comma into the sentence so that it translates to $(R \rightarrow C) \wedge J$.

(c) Complete the following truth table.

R	C	J	$R \rightarrow (C \wedge J)$	$(R \rightarrow C) \wedge J$
T	T	T		
T	T	F		
T	F	T		
T	F	F		
F	T	T		
F	T	F		
F	F	T		
F	F	F		

(d) Based off part (c), were your English teachers right to stress the importance of commas? Explain using the term **logically equivalent**.

(6) Find a (non-trivially) different statement which is logically equivalent to:

(a) $A \wedge (B \vee C)$

(b) $A \vee (B \wedge C)$

(c) $\neg(A \rightarrow B)$

(7) An advertisement for a tennis magazine says: “If I’m not playing tennis, I’m watching tennis. And if I’m not watching tennis, then I’m reading about tennis.” Assume the speaker can only preform one of these activities at a time. By translating into propositional logic, figure out what the speaker is doing.

HOMEWORK

(1) Assume that P is true, Q is false, and R is true. What is the truth value of

$$[(P \vee Q) \rightarrow \neg(\neg(P \wedge Q))] \leftrightarrow [Q \rightarrow (P \wedge R \leftrightarrow Q)]?$$

(2) Translate problems 5 and 6 from last weeks handout into propositional logic and solve them using truth tables.