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Review session

Problem 1

- *What is the number of inversions of the permutation corresponding to the following configuration of the 15 puzzle?
(Remember, the empty square is considered as the 16th tile.)*

1	2	3	4
	5	6	8
9	10	7	11
13	15	14	12

of inversions =

- What is the taxicab distance from the current position of the empty square to the lower-right corner of the puzzle?

$$d_{tc} =$$

- What is the value $\mathcal{P}(\mathcal{C})$ of the invariant \mathcal{P} for the configuration \mathcal{C} above?

$$\mathcal{P}(\mathcal{C}) =$$

- Is the above configuration \mathcal{C} of the 15 puzzle solvable? Why or why not?

Problem 2 Without doing any extensive computations, decide whether the following configuration of the 15 puzzle is solvable. Hint: compare this configuration to the one on page 9 (the problem continues to the next page).

1	2	3	4
	5	6	8
9	10	7	12
13	15	14	11

Explain your decision.

Problem 3 *Without doing any extensive computations, decide whether the following configuration of the 15 puzzle is solvable. Hint: compare it to the winning configuration.*

2	1	3	4
5	6	7	8
9	10	11	12
13	15	14	

Explain your decision.

Problem 4 Find the products $\delta \circ \sigma$ and $\sigma \circ \delta$ of the following two permutations.

$$\sigma = (3 \ 2 \ 4 \ 1) \quad \delta = (4 \ 3 \ 1 \ 2)$$

$$\delta \circ \sigma = (\quad \quad \quad)$$

$$\sigma \circ \delta = (\quad \quad \quad)$$

Do the permutations σ and δ commute?

Problem 5 The order of the permutation σ is 4. Find the following.

$$\sigma^{444}(7) =$$

Problem 6 For the transposition $\sigma = (52)$, find the following.

$$\sigma^{-1} =$$

$$\sigma^{2014} =$$

$$\text{sgn}(\sigma^{2015}) =$$

Problem 7 Find the order of the permutation $\sigma = (5\ 6\ 1\ 2\ 3\ 4)$.

The order of $\sigma =$

Without doing any additional computations, find the following.

$$\sigma^{-2} = \left(\quad \quad \quad \right)$$

Problem 8 Represent the transposition (85) as a product of adjacent transpositions.

$$(85) =$$

Problem 9

- Write down the full form of the permutation $\mu = (5\ 7\ 3)$ acting on a set of nine elements.

$$\mu = (\quad)$$

- Find the sign of μ .

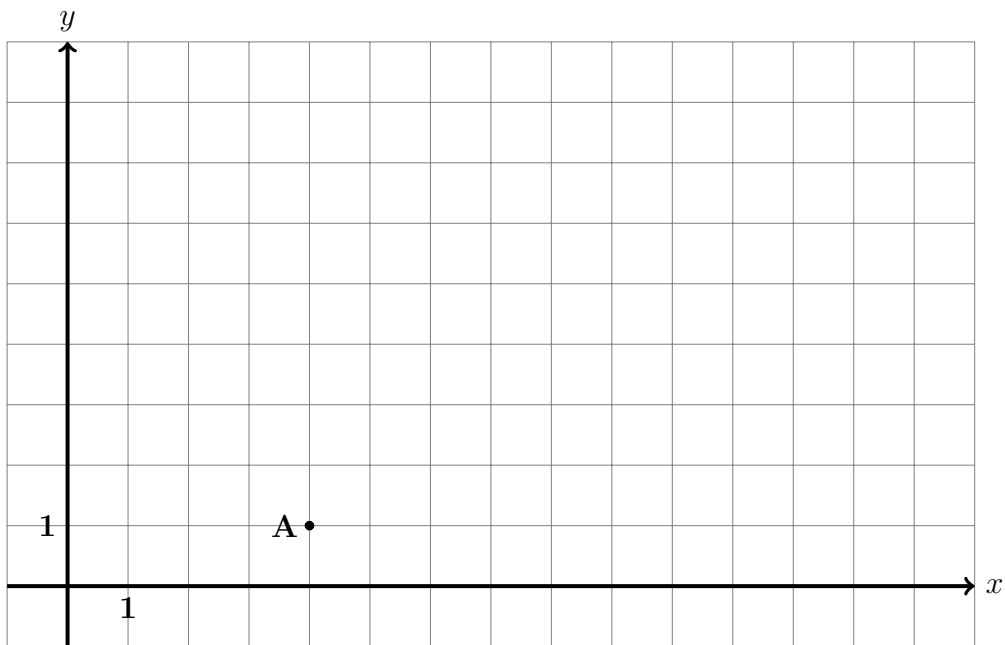
$$\text{sgn}(\mu) =$$

- Find μ^{-1} and write it down in the short form.

$$\mu^{-1} = (\quad)$$

Problem 10

- On the grid below, draw a point B such that $d_{tc}(A, B) > d_E(A, B)$.
- On the grid below, draw a point C such that $d_{tc}(A, C) = d_E(A, C)$.
- On the grid below, draw a point D such that $d_{tc}(A, D) < d_E(A, D)$. Is it possible? Why or why not?



Problem 11 Find the taxicab distance between the points $A = (p, q)$ and $B = (x, y)$.

$$d_{tc}(A, B) =$$

Find the taxicab distance between the points $C = (-3.3, 5.2)$ and $D = (-2.7, -7.8)$.

$$d_{tc}(C, D) =$$

Problem 12 For the configuration of the 15 puzzle below, find the taxicab distance between the squares 6 and 11.

1	2	3	4
5	6		8
9	10	7	11
13	15	14	12

$$d_{tc}(6, 11) =$$

Problem 13

- *What is the number of inversions of the permutation corresponding to the following configuration of the 15 puzzle?
(Remember, the empty square is considered as the 16th tile.)*

	2	3	4
1	5	6	8
9	10	7	11
13	15	14	12

of inversions =

- *What is the taxicab distance from the current position of the empty square to the lower-right corner of the puzzle?*

$d_{tc} =$

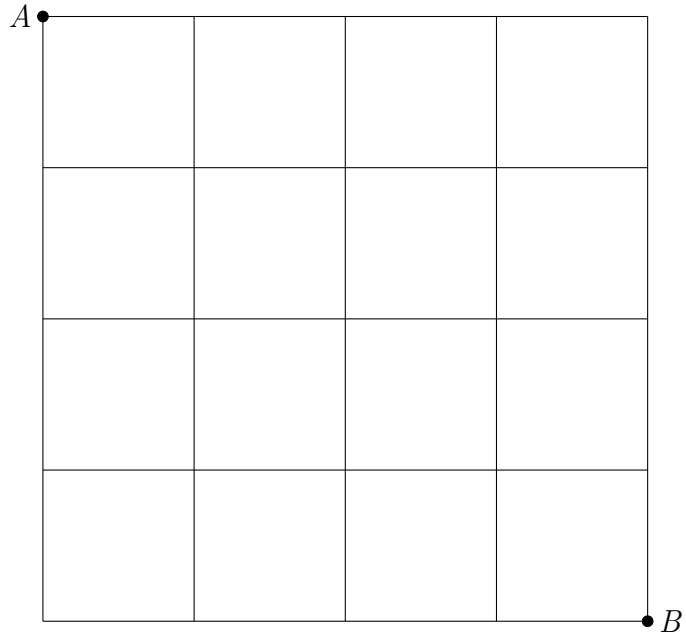
- *What is the value $\mathcal{P}(\mathcal{C})$ of the invariant \mathcal{P} for the configuration \mathcal{C} above?*

$$\mathcal{P}(\mathcal{C}) =$$

- *Is the above configuration \mathcal{C} of the 15 puzzle solvable? Why or why not?*

The review is over. If you have solved all the problems above and there still remains some time till the end of the class, please proceed to the next page.

Problem 14 *On the picture below, you are only allowed to move left/right or up/down along the grid lines. Find the number of all the shortest paths from A to B.*

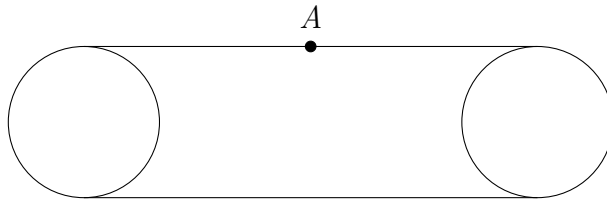


Problem 15 *There are 25 students at the UCLA Beginners' Math Circle. Is it possible for each student to have 7 friends in the class? Why or why not?*

Problem 16 *Without using a calculator, put the correct sign, $>$, $<$, or $=$ between the following numbers.*

$$300! \quad \sqrt{300^{300}}$$

Problem 17 *The picture below shows the rolling tracks of a bulldozer.*



As the bulldozer moves one inch forward, how many inches would the point A move? Why?

Problem 18 *Together Winnie the Pooh, Owl, Rabbit, and Piglet ate 70 carrots. Each ate a whole number of carrots and each ate at least one. The Pooh bear ate more carrots than each of the others. Owl and Rabbit together ate 45 carrots. How many carrots did Piglet eat?*

Problem 19 Find integral solutions x , y , and z for the following equation.

$$29x + 30y + 31z = 366$$

Problem 20 A certain country has several airfields. All the distances between the airfields are different. An airplane takes off from each of the airfields and flies to the closest airfield. Prove that at most 5 airplanes will land at each airfield.