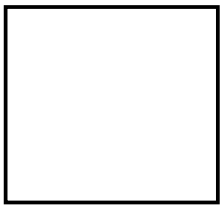
Planes, Tetrahedra, and Cross Sections

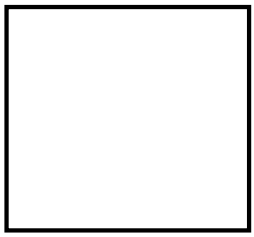
May 18, 2014

Warm Up Problems

1. Is it possible to cut a square into 7 smaller squares, not necessarily of equal size? If so, show how to do it by drawing the cuts on the square below. If not, explain why this is not possible.



2. You are cutting a square piece of paper. What is the largest number of pieces of paper you can get by making 3 straight cuts? The smaller pieces do not need to be squares. Show your answer on the square below.



3. What is the maximum number of pieces you can get by making 2 straight cuts on a cube? The smaller pieces do not need to be cubes.

4. What is the maximum number of pieces you can get by making 3 straight cuts on a cube? The smaller pieces do not need to be cubes.

Lines and Planes

There are three ways that two lines can interact with each other in a 3 dimensional system. Two lines can be parallel to each other, intersect each other, or be skew to each other.

- Two lines are parallel if you can draw a plane through them.
- Lines that intersect each other share one common point.
- Lines that are skew to each other if they do not lie in the same plane.

The figure below shows what these three scenarios look like.

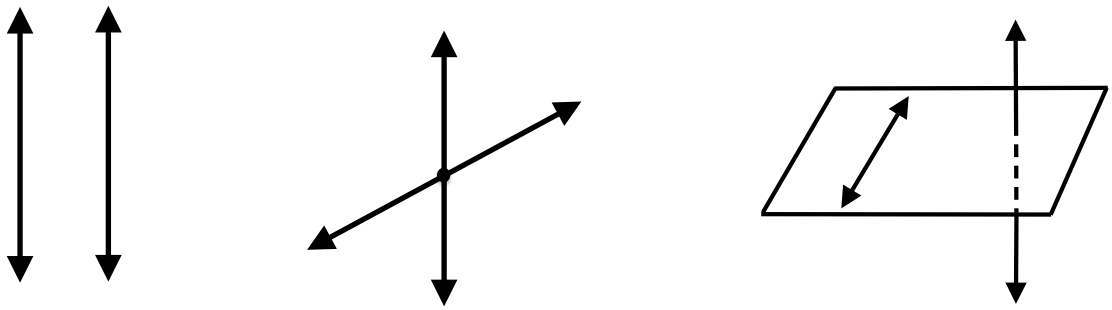


Figure 1: Parallel lines (left), intersecting lines (middle), and skew lines (right).

A plane is a flat, infinite, 2 dimensional surface. There are two ways that planes can interact with each other in a 3 dimensional system; they can be parallel to each other, or they can intersect. The figure below shows what these two scenarios look like. Note that even though planes are infinite, we draw boundaries on them to show their directions.

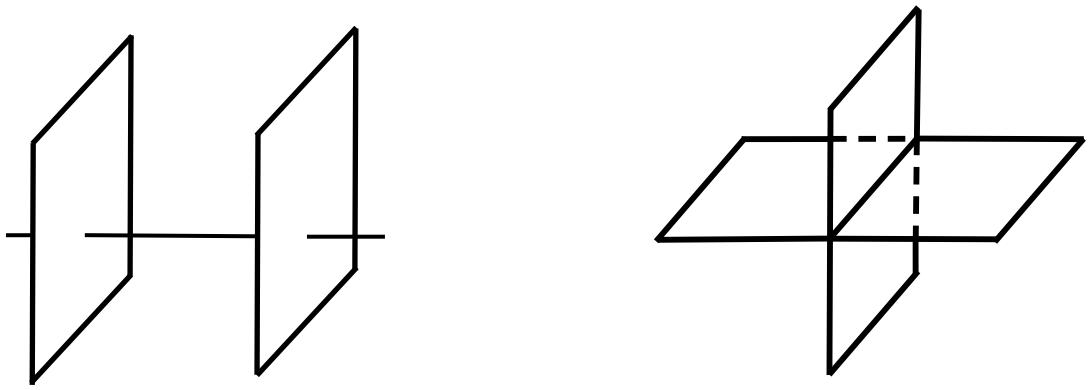


Figure 2: Parallel planes (left) and intersecting planes (right).

1. Answer the following questions about lines and planes.

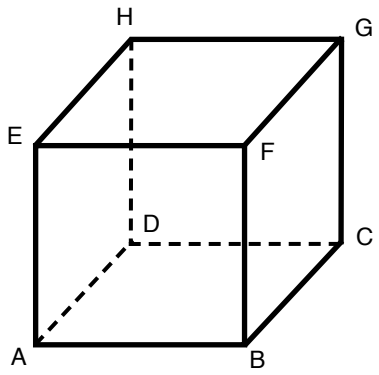
   (a) How many points are needed to define a line?

   (b) How many points are needed to define a plane?

   (c) What do two intersecting lines share?

   (d) What do two intersecting planes share?

2. Please indicate whether the following lines on the cube below are parallel, intersecting, or skew.



(a) AB and CD

(b) EH and EF

(c) CH and AB

(d) AH and BG

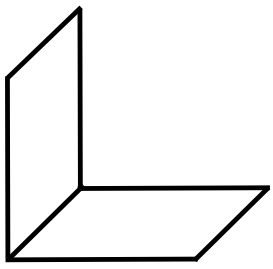
(e) AB and GC

(f) BH and AG

(g) BE and AH

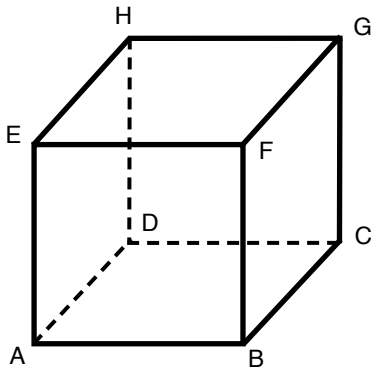
(h) BH and EG

3. To get practice in drawing planes, re-draw the two planes below. Then, extend the vertical plane downward, and the horizontal plane leftward.

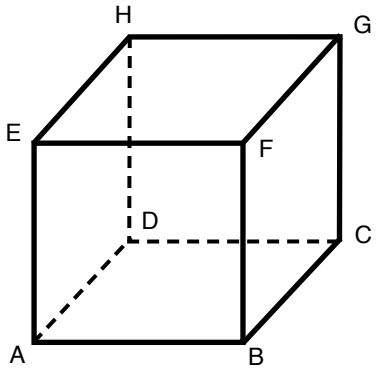


4. Draw the specified planes on the following cubes. Then, shade in the planes.

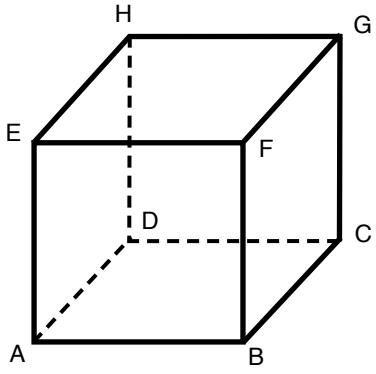
(a) Plane ABC



(b) Plane ABH



(c) Plane EDC

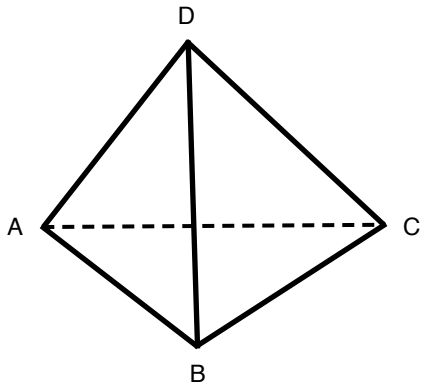


Tetrahedra and Cross Sections

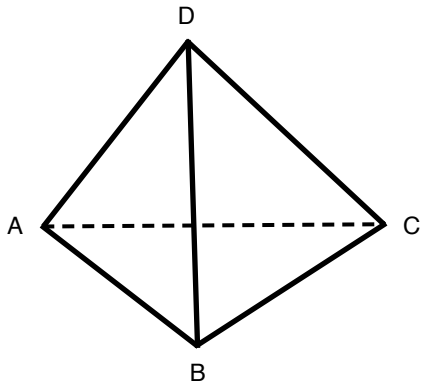
Tetra means 4 in Greek; thus a *tetrahedron* is a pyramid that has 4 faces.

1. What is the shape of the base of a tetrahedron?

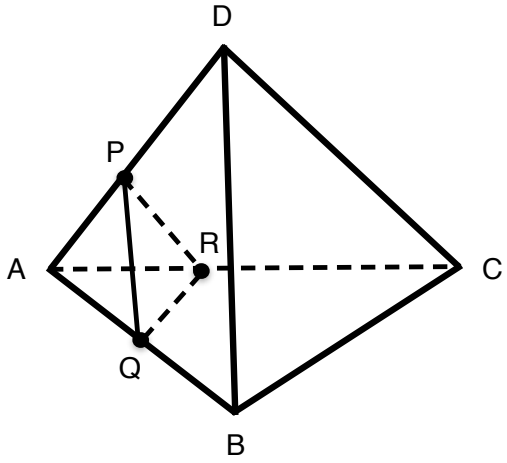
2. Below is a tetrahedron with vertices labeled A, B, C, D. Draw a plane that intersects the tetrahedron at vertex D. This plane and the tetrahedron should only share one common point (vertex D).



3. Draw a plane that intersects the tetrahedron at edge AD. This plane and the tetrahedron should only share edge AD.

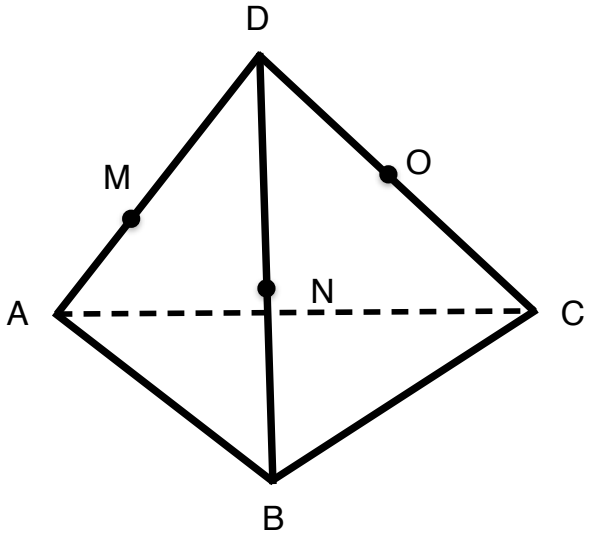


4. Planes can also "slice" through a tetrahedron. The shape that is made by the intersection of the plane and the tetrahedron is called the *cross-section.* Below is an example of a cross-section that is made by a plane intersecting a tetrahedron.



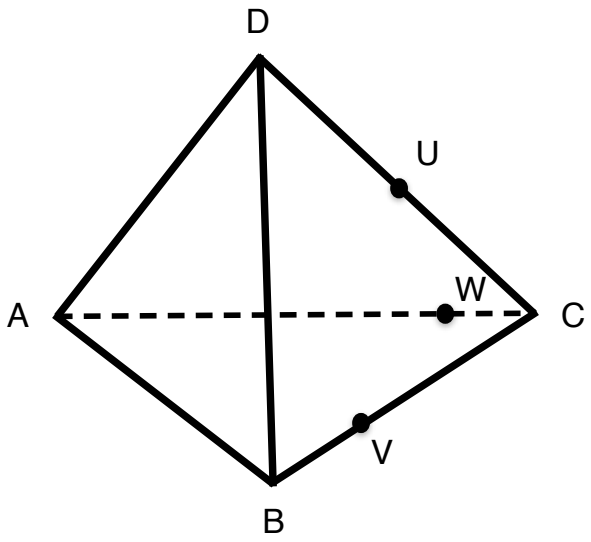
The cross-section that is created is called $\triangle PQR$. Please shade in $\triangle PQR$.

(a) Draw the cross-section of tetrahedron $ABCD$ below that is created by drawing a plane through the points M, N, and O. Shade in the cross-section.



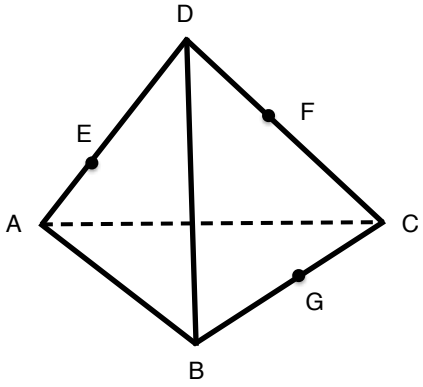
(b) What is the shape of this cross-section?

5. Draw the cross section that is made when a plane that contains points U, V, W intersects tetrahedron $ABCD$.



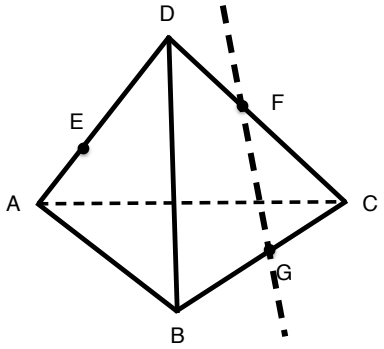
(a) What is the shape of this cross-section?

6. Drawing some cross sections are more difficult than the ones we have seen so far.

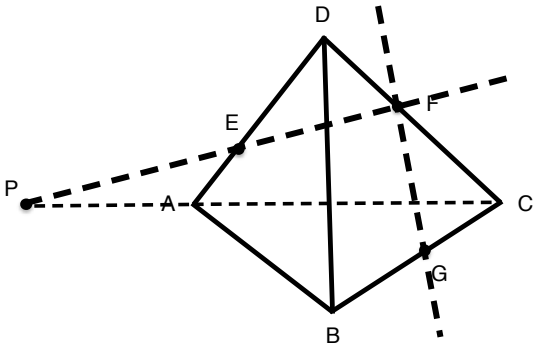


(a) Guess what the shape of the cross section is when tetrahedron $ABCD$ is intersected by plane EFG intersect.

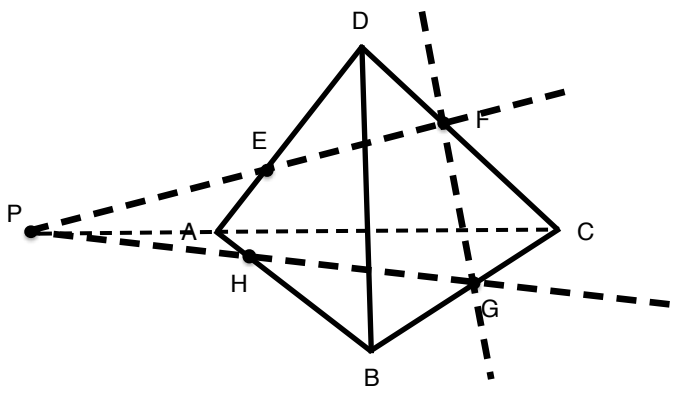
(b) To figure out what the cross section looks like, we need to extend the plane farther than just the triangle that is made by connecting points E, F, and G. First, we can draw a line that connects point F with point G.



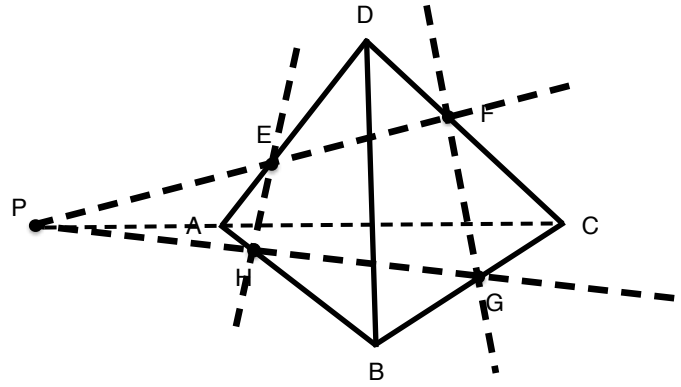
(c) Next, we draw the line that connects point E with point F. We can extend this line until it intersects with the line that connects point A with point C. We will call this point of intersection point P.



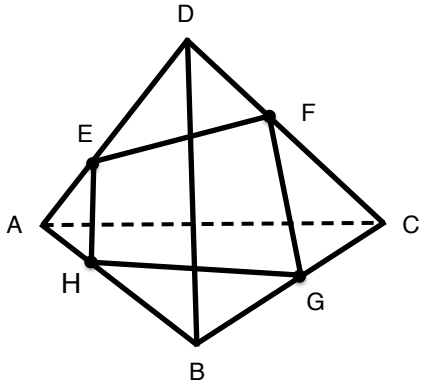
(d) Next, we draw the line that connects point G to point P. This line intersects edge AB at point H.



(e) We can connect points E and H.

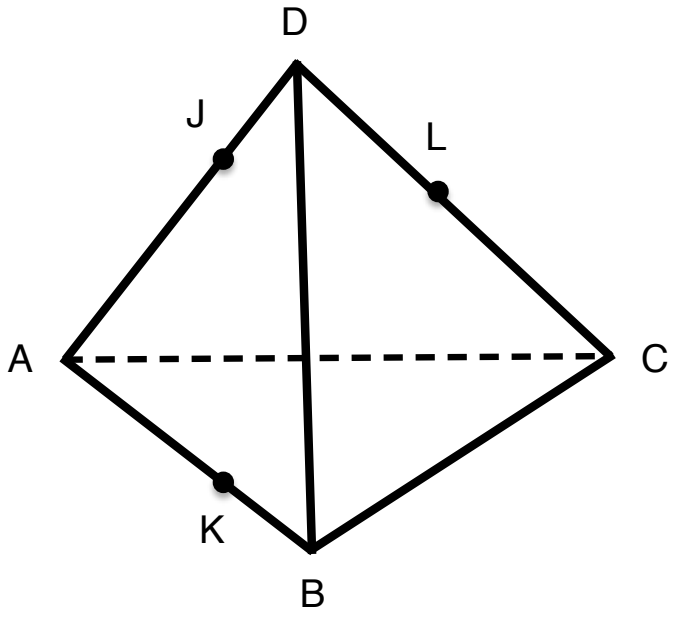


(f) Now, we can see what the cross-section looks like. Shade in the cross-section.



(g) What is the shape of the cross section?

7. Using the methods described previously, draw the cross-section that is made by the intersection of plane JKL and tetrahedron $ABCD$. Shade in the cross section.



What is the shape of the cross section?

8. What is the largest number of sides a cross-section of a tetrahedron can have? Explain your answer.

9. Below are tetrahedra. On the tetrahedra are points that are a part of the planes that intersect the tetrahedra. Draw the cross-sections of the tetrahedra and shade in the cross-sections.