Polyhedra

May 11, 2014

Warm Up Problem

Use modular arithmetic to explain a test of divisibility by 3. Use the number 3252 to help you explain the test.

$3252 = 3\times1000 + 2\times100 + 5\times10 + 2\times1$

$1000 \equiv 1 \pmod 3$

$100 \equiv 1 \pmod 3$

$10 \equiv 1 \pmod 3$

$1 \equiv 1 \pmod 3$

$3252 \equiv 3\times1 + 2\times1 + 5\times1 + 2\times1 \pmod 3$

$3252 \equiv 3+2+5+2 \pmod 3$

$3252 \equiv 12 \pmod 3$

we know that $12 \equiv 0 \pmod 3$

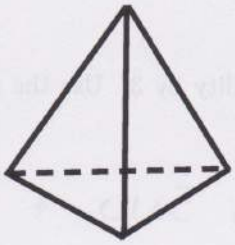
Therefore, $3252 \equiv 0 \pmod 3$

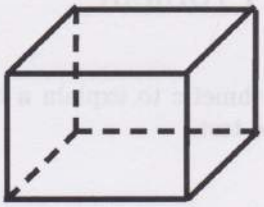
We see that if the sum of the individual digits is divisible by 3, then the number is divisibly by 3.

A polyhedron is a geometric 3-dimensional shape made up of several faces, straight edges, and vertices.

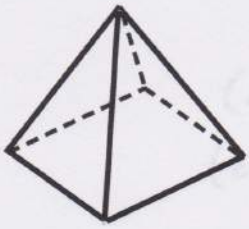
- A vertex is a point that describes the corners or intersections of geometric shapes.
- An edge is a line segment that connects two vertices.
- A face is a flat surface that forms part of the boundary of a solid object.

Below are some examples of polyhedra.





1. Answer the questions below about the following polyhedron.



(a) How many vertices are there?

5

(b) How may edges are there?

8

(c) How many faces are there?

5

2. Answer the following questions about polyhedra.

(a) Can a polyhedron have 3 vertices? Explain.

No. 3 vertices will define a two-dimensional surface, not a 3-dimensional shape.

(b) What is the smallest number of vertices a polyhedron can have?

4

(c) What is the smallest number of edges a polyhedron can have? Explain.

6. Each vertex has 3 edges attached to it, giving 12 edges. However, since two vertices share an edge, we have $12 \div 2 = 6$ edges.

(d) What is the smallest number of faces a polyhedron can have?

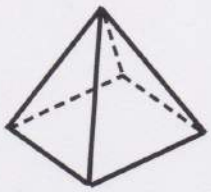
4.

Pyramids

A *pyramid* is a type of polyhedra that has the following properties:

- The base is a polygon.
- All the vertices of the base are connected with a special vertex called an *apex*.

Below is a picture showing an example of a pyramid.



3. Fill out the following table by writing how many vertices, edges, and faces pyramids with different bases have. Remember to count the apex as one of the vertices.

Base	Vertices	Edges	Faces
Triangle	4	6	4
Quadrilateral	5	8	5
Pentagon	6	10	6
Hexagon	7	12	7
n-gon	$n+1$	$2n$	$n+1$

(a) Using your answers from the table, complete the following relationship. Let v be the number of vertices, e be the number of edges, and f be the number of faces on a pyramid.

$$v - e + f = 2$$

(b) If a pyramid has 10 vertices, how many edges does it have? How many faces does it have?

$10 = n+1$

$n = 9$

edges $= 2n$

$= 2 \times 9$

$=$ 18 edges

faces $= n+1$

$= 9+1$

$=$ 10 faces

(c) If a pyramid has 20 edges, how many vertices and faces does it have?

$20 = 2n$

$n = 10 \Rightarrow$ Vertices $= n+1 \Rightarrow$ faces $= n+1$

$= 10+1$

$=$ 11 vertices

$=$ 11 faces

(d) A pyramid has F faces. How many edges does it have?

$F = n+1$

$n = F-1 \Rightarrow$ edges $= 2n$

$= 2F-2$

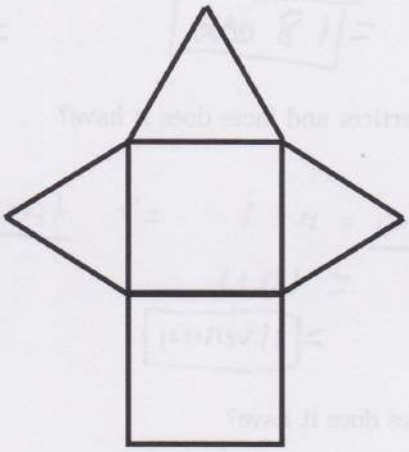
(e) Is it possible for a pyramid to have 2015 vertices?

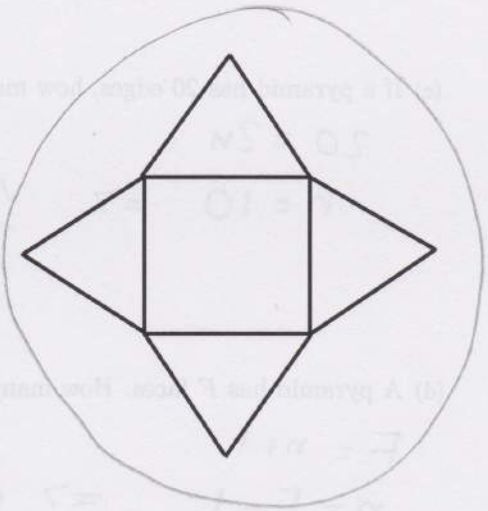
yes.

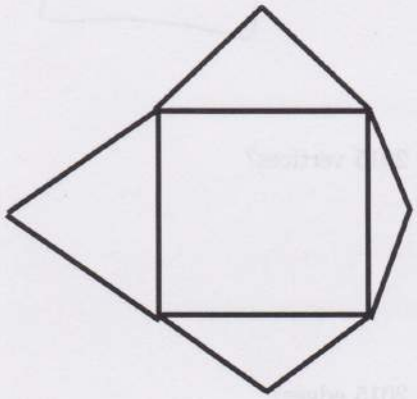
(f) Is it possible for a pyramid to have 2015 edges?

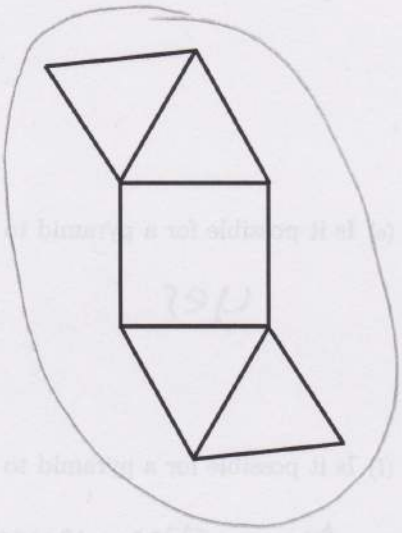
No. The number of edges must be even.

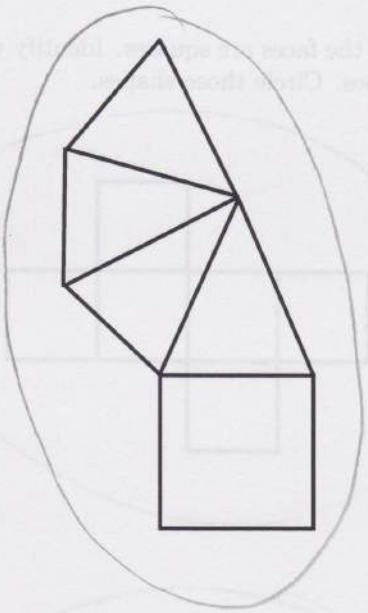
4. Identify which of the following 2-D shapes can be folded into pyramids. Circle those shapes.

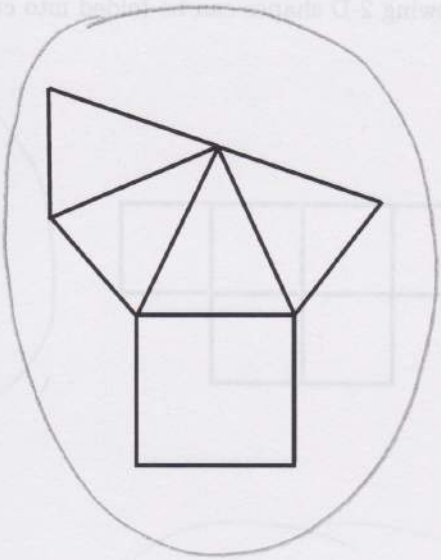


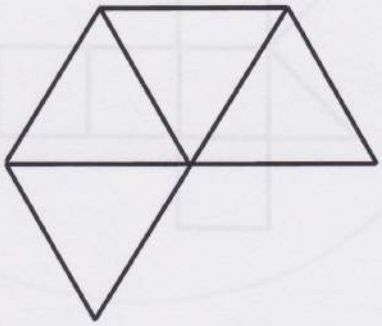


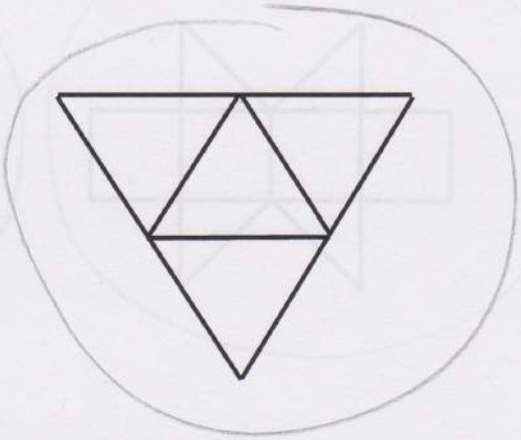


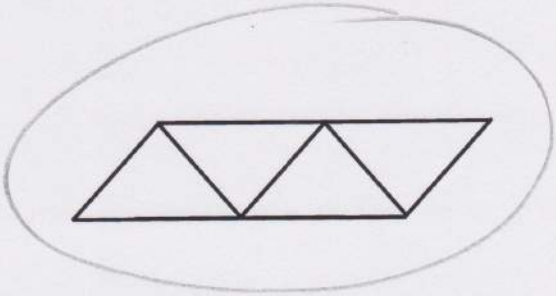




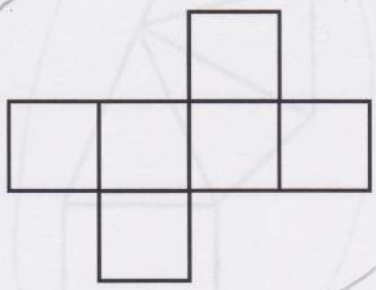


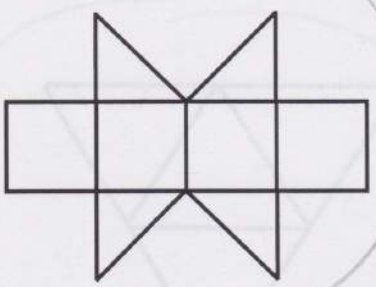


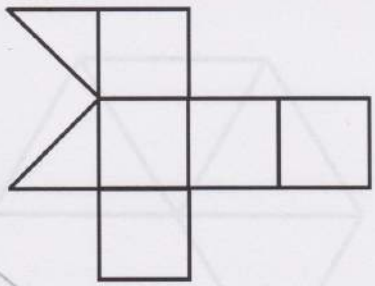




5. A cube is a special type of polyhedra where the faces are squares. Identify which of the following 2-D shapes can be folded into cubes. Circle those shapes.







6. A rectangular prism is a type of polyhedra where the faces are rectangles. Identify which of the following 2-D shapes can be folded into rectangular prisms. Circle those shapes.