LOGIC PUZZLES AND VECTOR REVIEW

All of the puzzles on this handout are well-known and have (for the most part) been communicated to me verbally. I do not claim to be the author of these puzzles, but no credit is given because in all cases the original author is unknown to me.

WARM UP

(Coin Flipping Game) Two people each flip a coin. Each person sees the result of his flip but not the result of his partners flip. Subsequently, each person must guess the result of the other persons flip. If either of the two people guess correctly the two people win. If neither guesses correctly they will be executed. Write down what you think is the pairs best strategy for survival. *(Note: Players cannot communicate after flipping their coins)*

PUZZLES

The following puzzles can all be solved by logical deduction. They are not arranged in order of difficulty, so you may solve them in any order you wish. Students will present solutions near the end of class.

(1) Dustin and Derek are at opposite ends of the country. Derek wants to send Dustin a birthday gift in the mail, but unfortunately the mailmen are very corrupt and will steal anything from a package that doesn't have a padlock on it. How can Derek get the contents of his package to Dustin? *(Note: Neither Dustin nor Derek have a padlock to which the other person has a key)*

(2) **(HARD)** Dustin comes from a town in Florida where each person wears either a red or blue beanie. If anyone figures out the color of her own beanie she must kill herself. Miguel came to visit Dustin and at a town meeting announced, "how interesting it is to see someone with a red beanie in this town." Determine Dustin's fate. *(Note: everyone in the town is perfectly logical and will make any valid logical deduction)*

(3) Morgan, Dustin, Derek, and Deven are trying to cross a rickety bridge in the middle of the night. Since the bridge is very rickety and dangerous no more than two people can cross it at a time and they must have a flashlight. Unfortunately, the group only has one flashlight. Find a way for the group to cross the bridge in 17 minutes assuming each person moves as follows:

- Dustin takes 1 minute to cross the bridge.
- Derek takes 2 minutes to cross the bridge.
- Morgan takes 5 minutes to cross the bridge.
- Deven takes 10 minutes to cross the bridge.

(4) Miguel owed Morgan some money and paid him in coins. Miguel gave Morgan 12 coins, but one of them was counterfeit (and hence had a different weight than the other coins). Morgan doesn't know if the false coin is heavier or lighter than the other coins. How can Morgan find the false coin using three weighs on a simple scale?

(5) You and two other people are each given a playing card. You can see the other two people's cards but not your own. Anyone who sees at least one red card must raise their hand. You see two red cards and raise your hand. Both of the other people also raise their hands. Subsequently, everyone is given a chance to state the color of their card if they know it. Everyone stays silent. Thirty seconds later everyone is given another chance to state the color of their card if they know it. This time you answer. What is your answer? *(Note: the other two people are assumed to be perfect logicians and make any valid logical deduction)*

(6) You arrive at Math Circle early and there is a sandwich on the table. You're very hungry so want to eat it, but it might be poisoned. Deven, Dustin, and Derek are already in the room before you and you know that each of them will either always lie or always tell the truth. You ask, "are you truth-tellers?" Deven responds but you can't hear him, so you ask what he said. Dustin replies, "He said he was a truth-teller." And Derek says, "No, he said he was a liar." How can you determine whether or not the sandwich is edible?

(7) **(HARD)** The situation is the same as problem 6. However, now one of the three instructors always lies, one always tells the truth, and one answers perfectly randomly. You can ask two yes or no questions, each of which must be directed at exactly one of the three instructors. How can you determine whether or not the sandwich is edible?

(8) After a lot of petitioning, the UCLA math department decided to give every student a locker. There are 1000 lockers in total and they have been numbered from 1 through 1000. During recess (of course UCLA has recess!), the students decide to try an experiment. When recess is over each student will walk into the school one at a time. The first student will open all of the locker doors. The second student will close all of the locker doors with even numbers. The third student will change all of the locker doors that are multiples of 3 (change means closing lockers that are open, and opening lockers that are closed.) The n-th student will change the orientation of all locker doors that are multiples of n. After 1000 students have entered the school, how many locker doors will be open?

(9) A deck has 52 cards, 14 of which are face up. You are in a dark room with the deck (so you cannot see the individual cards). Can you produce two stacks of cards which each have the same number of cards face up?

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VECTOR REVIEW (HOMEWORK)

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This section is considered homework, but you may do it early if you finish the rest of the handout. Solve the following problems. Students will present solutions at the beginning of next week.

(1) Suppose we have a triangle ABC. Let M_A be the midpoint of side BC, M_B be the midpoint of side AC, and M_C be the midpoint of side AB. Express the following in terms of the vectors $\vec{v} = \overrightarrow{AB}$ and $\vec{w} = \overrightarrow{AC}$. *(DRAW A PICTURE!!)*

    (a) $\overrightarrow{AM_A}$

    (b) $\overrightarrow{BM_B}$

    (c) $\overrightarrow{CM_C}$

(2) The notation from the previous problem continues here. Compute each of the following in terms of $\vec{v}$ and $\vec{w}$:

(a) $\frac{2}{3}\overrightarrow{AM_A}$

(b) $\vec{v} + \frac{2}{3}\overrightarrow{BM_B}$

(c) $\vec{w} + \frac{2}{3}\overrightarrow{CM_C}$

(3) Prove that all three medians of a triangle intersect at a single point.

(4) (Midpoint theorem) **(HARD)** Let ABC be a triangle. Let M be the midpoint of AB and N be the midpoint of AC. Show that the length of MN is half the length of BC. *(Hint: Express each side of the triangle with vectors and show that $\overrightarrow{MN} = \frac{1}{2}\overrightarrow{BC}$!)*