LAMC Beginners' Circle **May 11, 2014**

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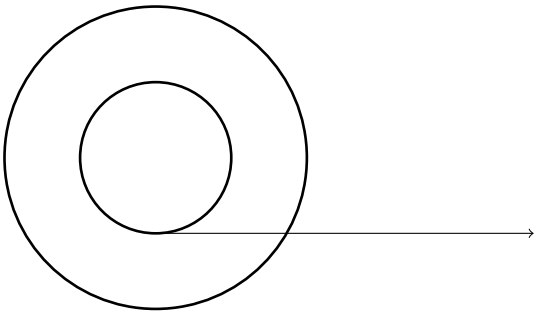
Problem solving session

Problem 1 *The roll of thread pictured below has the inner radius of 1 cm and the outer radius of 2 cm.*



The roll is rolling over the table at the speed of 30 cm/s. What is the speed of the thread that is pulling the roll?

Problem 2 *Two boys of equal mass initially stand still on the ice of a skating rink holding the opposite ends of a rope of length l. The boys start pulling the rope. As a result, the first boy moves towards the second at the speed v. Moving his hands faster, the second boy moves towards the first at the speed $3v$. What time would it take the boys to meet? In what ratio would the meeting point divide the original distance l between the boys?*

Problem 3 *What number should one add to both the numerator and denominator of the fraction* $11/41$ *to obtain the fraction* $3/8$*?*

Problem 4 *An isosceles triangle has two sides of length 4 units and the third side that is 6 units long. Is the triangle acute, right, or obtuse? Please give some detailed reasoning to support your decision.*

Problem 5 *The straight lines extending the sides AB and CD of a quadrilateral $ABCD$ meet at the 20° angle. The same is true for the straight lines BC and AD. Prove that the quad has two angles of equal size and two more angles that are different by 40°.*

Problem 6 *Find all the four-digit numbers that are perfect squares and have the first two as well as the second two digits equal.*

Problem 7 *A square is cut into five rectangles. Four of the rectangles are congruent. Each of the congruent rectangles shares a common angle with the square. The fifth rectangle is located inside the square (has no common points with the sides of the square). Prove that the fifth rectangle is itself a square.*

Problem 8 *The numbers 1, 2, 3, ... ,2014 are written on the board. First, Arul erases every tenth number, i.e. the numbers 10, 20, 30, etc. Then he erases every ninth number, followed by every eighth, seventh, sixth, fifth, fourth, third, and second. How many numbers are left on the board? What is the number in the last position?*

Problem 9 *All natural numbers are painted either black or white. Prove that there exist the numbers A, B, and C such that all three have the same colour and $A + B = 2C$.*