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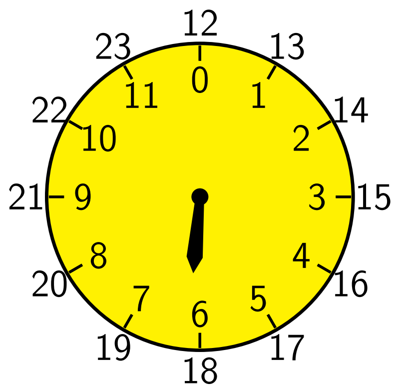
Problem 1 *It takes a grandfather's clock 30 seconds to chime 6 o'clock. Assuming that the time of each chime is negligible compared to the time intervals between the chimes, how much time would it take the clock to chime 12?*

Clock Arithmetic or a Circle as a Number Line

One way to turn a circle into a number line is to divide it into twelve equal parts. In this case, one step is usually called one hour.



0 coincides with 12. The hour hand moves from 0 to 1, from 1 to 2, ... from 11 to 12 just as it would have on the straight number line. However, 12 equals 0 on this circle, so there it goes again, from 1 to 2, and so on. We write down the fact that 12 equals 0 as

$$12 \equiv 0 \ (mod\ 12) \tag{1}$$

and read it as *12 is congruent to 0 modulo 12*. The usual "=" sign is reserved for the straight number line; we use "≡" on the circle instead. The *mod* 12 symbol tells us that the circle is divided into 12 equal parts, so 12 coinsides with 0, 13 – with 1, 14 – with 2, and so on. Or in the new notations,

$$13 \equiv 1 \ (mod\ 12), \ \ 14 \equiv 2 \ (mod\ 12), \ldots, 23 \equiv 11 \ (mod\ 12),$$

$$24 \equiv 12 \equiv 0 \ (mod\ 12).$$

Problem 2 *Divide the following numbers by 12, and write the remainders of the divisions.*

15	*21*
37	*46*
80	

Now, write down the modular congruencies for the following numbers.

$$15\ (mod\ 12) \equiv$$

$$21\ (mod\ 12) \equiv$$

$$37 \ (mod\ 12) \equiv$$

$$46 \ (mod\ 12) \equiv$$

$$80 \ (mod\ 12) \equiv$$

Problem 3 *Write down the modular congruencies for the following additions.*

$$9 + 4 \equiv \qquad\qquad (mod\ 12)$$

$$18 + 8 \equiv \qquad\qquad (mod\ 12)$$

When subtracting two numbers ‘a‘ and ‘b‘, finding a - b means finding a number ‘c‘ such that c + b = a.
For example, 5 - 3 =2 because when you add 2 to 3, you get 5.

The same occurs with modular arithmetic.

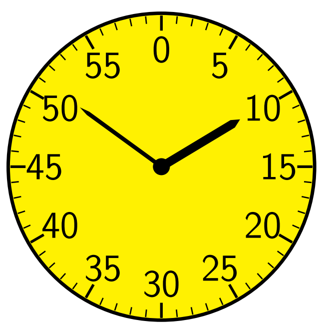
Write down the modular congruencies for the following subtractions.

$$8 - 3 \equiv \qquad\qquad (mod\ 12)$$

$$1 - 11 \equiv \qquad\qquad (mod\ 12)$$

$$4 - 15 \equiv \qquad\qquad (mod\ 12)$$

Another standard way to turn a circle into a number line is to divide it into 60 equal parts. Depending on the situation, the unit step is called either a minute or a second.



All the numbers living on this number line are considered modulo 60. In particular, $60 \equiv 0\ (mod\ 60)$. There are 60 minutes in an hour.

Problem 4

$72 \equiv \qquad\qquad (mod\ 60)$

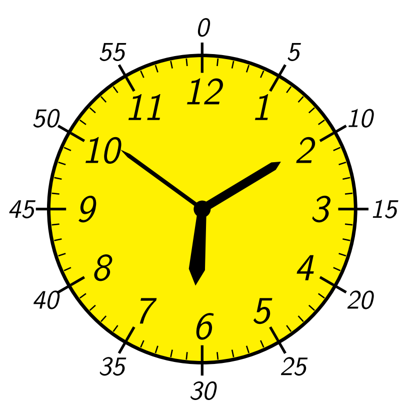
$135 \equiv \qquad\qquad (mod\ 60)$

$55 + 55 \equiv \qquad\qquad (mod\ 60)$

$-15 \equiv \qquad\qquad (mod\ 60)$

$240 - 59 \equiv \qquad\qquad (mod\ 60)$

Problem 5 *What is the time, in hours, minutes, and seconds, on the clock below?*



There are 24 hours in a day, so one more standard way to turn a cricle into a number line is to divide it into 24 equal parts. The US military use the 24-hour clock. The following is a photograph of the 24-hour clock from the USS (United States Ship) *Mullinnix*, the last "all gun" US Navy destroyer in the Pacific, decommissioned in 1982.[1]

USS *Mullinnix* 24-hour clock.[2]

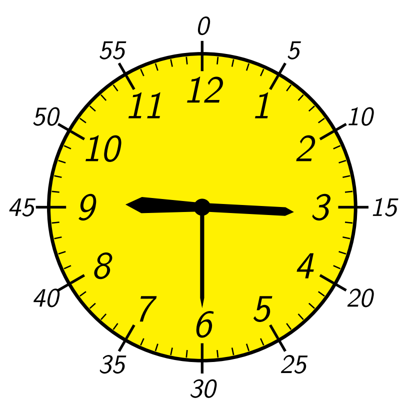
[1]See its homepage at `http://www.ussmullinnix.org/`

[2]Downloaded from `http://www.ussmullinnix.org/MuxMemorabilia.html`

Since $60 \div 24$ is not a whole number, we can't use the same marks on the face of a 24-hour clock for minutes and hours (to better see this, please find the minute and hour marks on the face of the USS *Mullinnix* clock). $60 \div 12 = 5$, so this inconvenience doesn't exist for the clocks and watches we are used to. On the other hand, to disambiguate between, say, 1 o'clock night time and 1 o'clock afternoon, we have to use the A.M./P.M. notation not needed in the military. In their language, 1 o'clock P.M. is 13:00 (thirteen hundred) hours, plain and simple.

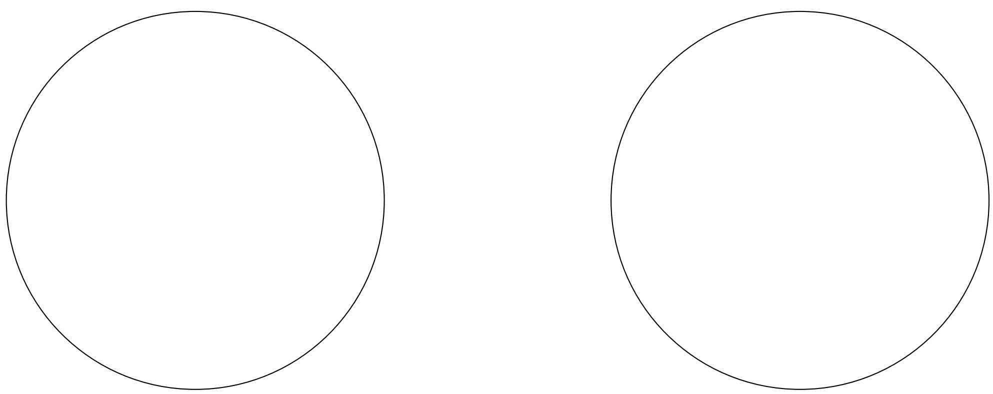
Problem 6 *What time does the USS Mullinnix clock show?*

Problem 7 *What is the time on the clock below?*



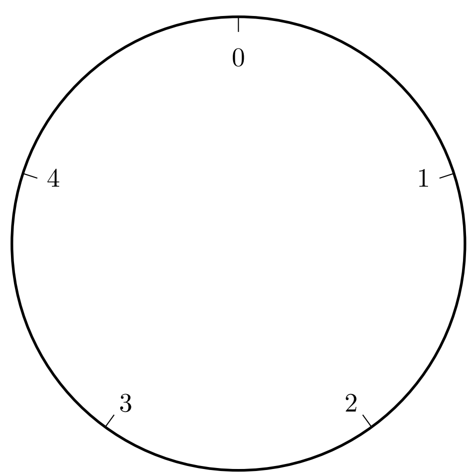
Suppose that this is the time P.M. How would the military call it?

Problem 8 *On the left, draw the civilian clock showing 1:45. On the right, draw the military clock showing the same time P.M.*



Problem 9 *An experiment in a biological lab starts at 7:00 AM and runs for 80 hours. What time will it end?*

In the following problems, we will consider the *mod* 5 arithmetic, that of a circle divided into five equal parts.



Problem 10 *Writw down the modular congruencies for the following multiplation problems.*

$$2 \times 3 \equiv \qquad\qquad (mod\ 5)$$

$$4 \times 4 \equiv \qquad\qquad (mod\ 5)$$

$$5 \times 7 \equiv \qquad\qquad (mod\ 5)$$

Similar to how subtraction is related to addition, division is related to multiplcation. Finding $a \div b$ means finding a number 'c' such that
$b \times c = a$.

For example, $6 \div 3 = 2$ *because* $3 \times 2 = 6$.

Write down the modular congruencies for the following division problems.

$$1 \div 2 \equiv \qquad\qquad (mod\ 5)$$

$$1 \div 3 \equiv \qquad\qquad (mod\ 5)$$

$$1 \div 4 \equiv \qquad\qquad (mod\ 5)$$