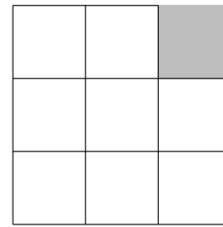
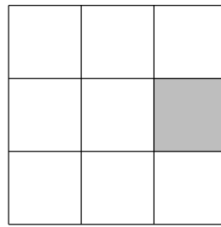
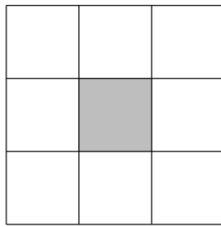


Life in an Insect World¹

April 6, 2014

Warm Up Problems

1. A water lily growing in a pond doubles in size every day. If it takes 60 days for the lily to cover the entire pond, how long does it take to cover half of the pond?
2. Two pieces were used to make each of the three white shapes below. The grey squares are not a part of the white shapes. Draw the two pieces that were used.



3. Place the digits 0 through 9 such that the following addition creates the largest sum.

$$\square + \square\square + \square\square\square + \square\square\square\square$$

4. Now place the digits 0 through 9 such that the following multiplication creates the largest product.

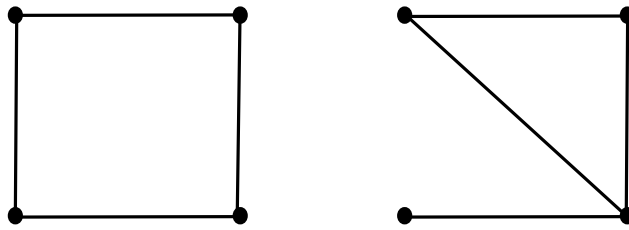
$$\square \times \square\square \times \square\square\square \times \square\square\square\square$$

¹Adapted from D. Farmer, T. Stanford, "Knots and Surfaces", Chapter 1.

Mapping the Insects' Countries

Each country in an insect world consists of several cities. Some cities are connected by tunnels. The insects are somewhat intelligent, but are unable to measure distances or determine direction of movement. They know from their travels which cities are connected by roads and which are not. The insects are smart enough not to build two or more tunnels connecting the same pair of cities, or to build a tunnel connecting a city with itself. However, the insects do not have maps of their own countries.

1. Here are the maps of the Antland and Beeland (see pictures below). Only you have these maps. The insects do not have them.

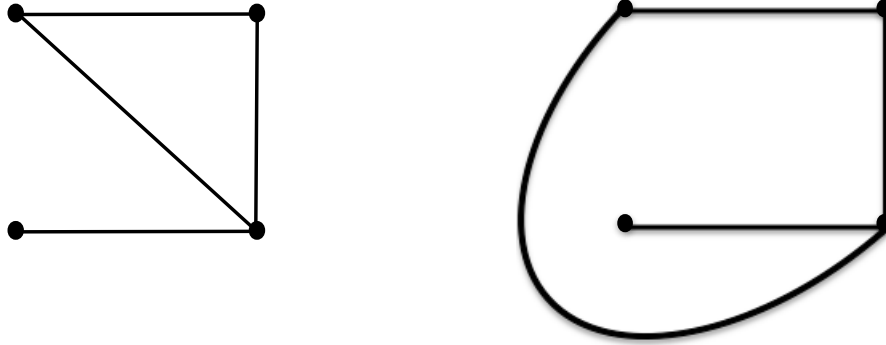


The insects of Antland talk over the phone with the insects of Beeland. They want to decide if their countries look the same. They start by asking

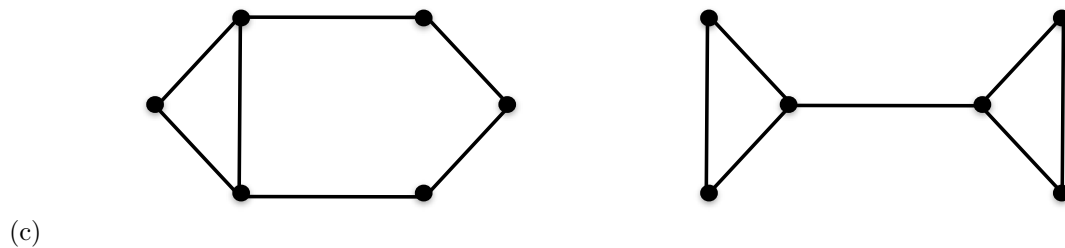
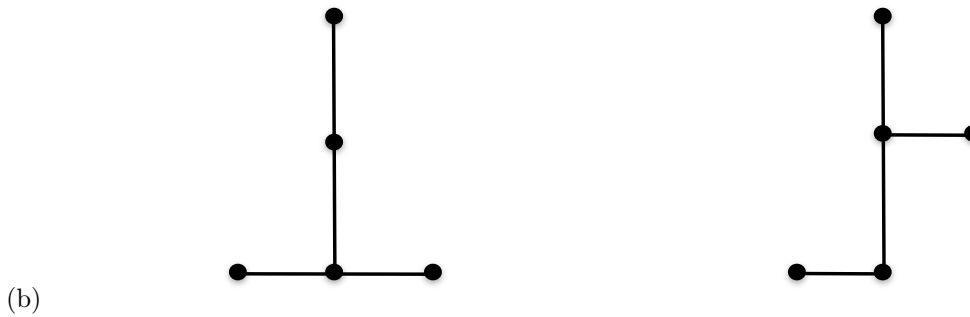
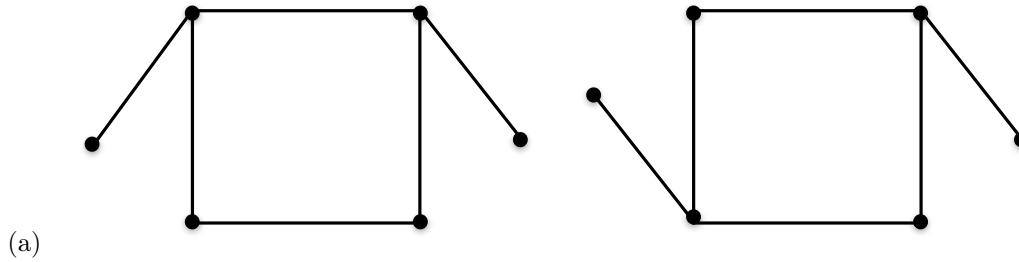
How many cities do you have in your country?

What questions can they ask next in order for them to decide whether their countries “look the same”?

2. Can the insects distinguish between the two countries below? Why or why not?



3. For each pair of countries below, decide whether insects would view them as the same or different. Notice that in all examples the number of cities and the number of tunnels are the same for both countries.



4. Quadroland has 4 cities. Draw all possible ways tunnels can join the cities in Quadroland. (Remember that some cities might not be connected to each other).

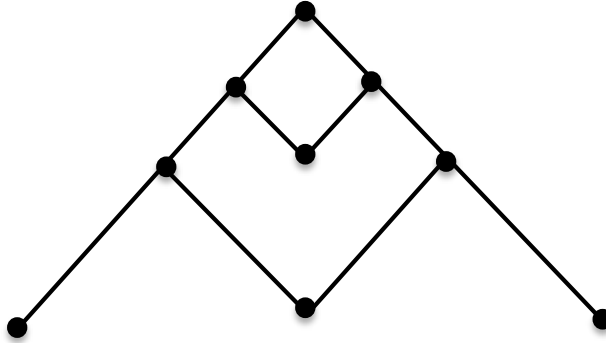
5. A wasp says that:

- there are 7 cities and 9 tunnels in its country;
- one city has just one tunnel connected to it;
- one city has five tunnels connected to it;
- two cities have three tunnels connected to them;
- the other three cities have two tunnels connected to them.

Draw a map that fits this description. Then, draw another one. How can you tell these two countries apart?

6. In one of the countries, each city has a tower. Insects decided to decorate the cities by putting stars on the towers in the following way: the number of stars they put on a tower equals to the number of tunnels connected to its city.

(a) For the country below, write the number of the stars next to each of the cities.



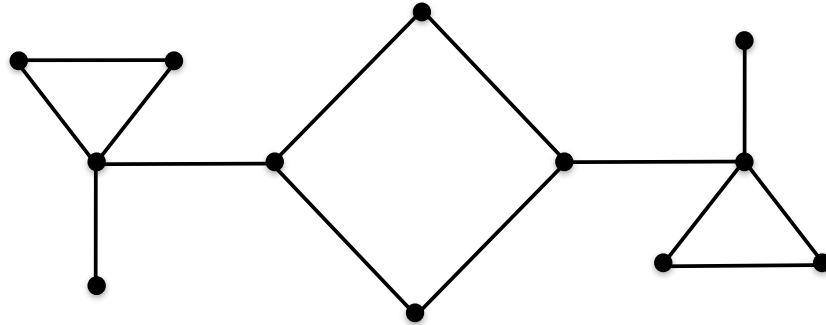
(b) What is the total number of stars in this country?

(c) How is the total number of stars and the number of tunnels in the country related?

(d) Can you explain this? Do you think this is always going to be the case?

7. The insects noticed that some of their cities have an even number of stars. They decided to call these cities “Even Cities”. All of the other cities were called “Odd Cities”.

(a) How many Odd Cities does the county below have?



(b) Draw your own insect country. Write down the number of stars for each city. How many Odd Cities does your country have? How many Even Cities does it have?

8. Insects noticed that in all of their countries

The number of Odd Cities is even.

Let's help the insects prove their statement.

- (a) Assume that a country does not have any tunnels. How many stars are there on each city? What can you say about the number of Odd Cities at this moment (is it even or odd)?

- (b) Suppose that at some moment, insects build a new tunnel which connects two Odd Cities. Is the number of Odd Cities even or odd?

- (c) Suppose that insects build a new tunnel which connects two Even Cities. Is the number of Odd Cities even or odd?

- (d) Suppose that the insects build a new tunnel which connects an Odd City with an Even City. How does this change the number of Odd cities?

- (e) Can you put together your observations in parts (a)-(d) to draw a conclusion about the number of Odd Cities?