LAMC Beginners' Circle **April 20, 2014**

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Problem 1 *Use arithmetic operations $+$, $-$, $\times$, and $\div$ as well as parenthesis to turn the following formulas into correct equalities.*

$$4 \quad 4 \quad 4 \quad 4 = 5$$

$$4 \quad 4 \quad 4 \quad 4 = 17$$

$$4 \quad 4 \quad 4 \quad 4 = 20$$

$$4 \quad 4 \quad 4 \quad 4 = 32$$

$$4 \quad 4 \quad 4 \quad 4 = 48$$

$$4 \quad 4 \quad 4 \quad 4 = 64$$

Problem 2 *Find eight consecutive integers such that the sum of the first three of them equals to the sum of the last five.*

Problem 3 *Find the largest natural (positive integral) number such that*

a. all of its digits are different;

b. all of its digits are different and the number is a multiple of four.

Problem 4 *There are nine grapplers on a schools grappling team, all of different skill. A more skilful athlete always wins a match against a less skilful one. The coach wants to split the grapplers into three practice teams, three athletes in each, so that when each athlete grapples with all others that are not in his team, the first team beats the second, the second team beats the third, and the third team beats the first. If you think this is possible, please help him do so.*

Problem 5 *Oleg gave his class a hard math problem. The number of the boys who solved the problem turned out to be equal to the number of the girls who didn't solve it. Is the number of all the kids who solved the problem greater, less, or equal to the number of all the girls in the class?*

Problem 6 *John took a cup full of coffee, drank 1/6 of it and filled up the cup with milk. Then he drank 1/3 of the mix and filled it up with milk again. Then he drank 1/2 of the cup and filled it up with milk one more time. Finally, he drank the entire cup. What did John drink more, coffee or milk?*

Problem 7 *It is known that the numbers a and b are integers and that the fraction*

$$\frac{a-b}{a+b}$$

can be simplified ($a-b$ and $a+b$ have a common factor). Can the fraction

$$\frac{a}{b}$$

be simplified? Why or why not?

Problem 8 *Solve the following equation.*

$$2014 = 1 + \cfrac{1}{1 - \cfrac{4}{1 + \cfrac{10}{1 + \cfrac{32}{1 + \cfrac{7}{1 - \cfrac{3}{1 + \cfrac{17}{x}}}}}}}$$

Problem 9 *A student wrote two numbers, first and second, on a paper sheet. He obtained the third number as the sum of the first and second. He further obtained the fourth number as the sum of the second and third and proceeded this way to get the fifth and sixth number. What is the sum of the six numbers if the fifth number equals seven?*

Problem 10 *A store manager increased the price of an item 100%. The item didn't sell well at the new price, so the manager decreased it 50%. Which price is greater, the original or the final?*

Problem 11 *The number of boys attending a Math Circle is less than 50% but more than 40% of all the students at the Circle. What is the smallest possible number of students at the Circle?*

Problem 12 *Solve the following cryptarithm.*

$$\begin{array}{cccccc} & & T & H & I & S \\ + & & & & I & S \\ \hline & E & A & S & Y \end{array}$$

Problem 13 *Without switching to the decimals, find the hexadecimal representation of the following octal number.*

$$5267_8$$

$$5267_8 =$$

Problem 14 *Recall that the mod* 5 *arithmetic is that of a circle divided into five equal parts. Compute the following.*

$$2 \times 3 \equiv \qquad (mod\ 5)$$

$$1/2 \equiv \qquad (mod\ 5)$$

$$3^{2014} \equiv \qquad (mod\ 5)$$

Problem 15 *Find the order of the permutation* $\sigma = \begin{pmatrix} 3 & 5 & 1 & 2 & 4 \end{pmatrix}$.

Problem 16 *For the permutation* σ *from the previous problem, find* σ^{2014}.

$$\sigma^{2014} = \begin{pmatrix} & & & & \end{pmatrix}$$

Problem 17 *Represent σ from Problems 15 and 16 as a product of transpositions.*

Problem 18 *Find a seven-digit number with all the digits different such that the number is divisible by the product of its digits.*